ISSN 2959-6130

Comprehensive Analysis of Portfolio Optimization Using the Markowitz and Index Models

Ziye Luo

Abstract:

Harry Markowitz's 1952 systematic approach to optimizing the trade-off between risk and return, known as Modern Portfolio Theory (MPT), revolutionized investment strategy. The objective of this final project is to apply the Markowitz and index models to provide a comprehensive analysis of portfolio optimization and conduct a comparative examination of the two models. By utilizing 20 years of historical daily total return data for 10 selected stocks within the S&P 500 Index, the study will develop optimal portfolios under various realistic constraints, such as typical leverage limitations and short selling restrictions. Additionally, the study evaluates the performance of these models and demonstrates that while index models offer computational simplicity, there are some drawbacks. In this scenario, a Markowitz model will generally provide a more precise risk-return optimization. Finally, the findings highlight the substantial influence of real-world constraints on portfolio management, which can significantly alter the efficient frontier and the structure of the optimal portfolio. This article, in the chapter on future research, proposes some possible future research directions, including the integration of behavioral finance elements and dynamic portfolio optimization, and furthermore discusses the implications of these research results.

Keywords: Portfolio Optimization, Markowitz Models, Modern Portfolio Theory (MPT), Index model

1. INTRODUCTION

Modern Portfolio Theory (MPT), initially introduced by Harry Markowitz in 1952, is a fundamental framework for constructing portfolios that effectively balance risk and return and is widely utilized by institutional investors such as mutual funds, pension funds, and hedge funds. The basic concept is that investors can build a portfolio by diversifying their investments into various assets and optimizing the portfolio to achieve the maximum potential return for which the investor is willing to associate the risk. In the same case, risk is determined by the standard deviation of historical returns of a selected asset, which is indicative of its volatility.

This article examines portfolio optimization by applying the Markowitz model (MM) and the Index model (IM), both of which are essential components of Modern Portfolio Theory (MPT). The analysis was conducted on a dataset comprising 10 distinct sectors of stocks' 20-year historical returns along with the S&P 500 Index. Among them, NVDIA Corporation, Cisco Systems, Inc., and Intel Corporation are stocks that belong to the technology sector, while The Goldman Sachs Group, Inc, U.S. Bancorp, The Toronto-Dominion Bank, and The Allstate Corporation belong to the financial service sector, The Procter & Gamble Company and Colgate-Palmolive Company belong to Consumer Defensive, and Johnson & Johnson belongs to healthcare type. (Abbreviated in the chart as NVDA, CSCO, INTC, GS, USB, TD CN, ALL, PG, JNJJ, CL.)

Five different constraint scenarios that mirror real-world obstacles in investing challenges in the analysis. These scenarios include limitations imposed by Regulation T, restrictions on maximum asset weight, absence of any constraints, restrictions on short sales, and beta-weighted portfolio constraints. The aim is to assess the extent to which these constraints influence the formation of optimal investment portfolios and to compare the effectiveness of Markowitz and index models in attaining optimal risk-adjusted returns. Simultaneously, the analytical results of both models for the ideal investment portfolio in terms of risk are compared, a research analysis is conducted using relevant literature, and the application of future investment portfolios is prospected and referenced.

2. LITERATURE REVIEW

Throughout its history, MPT has implemented significant modifications to its investment strategy and overall market portfolio decisions. While MM and IM offer a certain level of mathematical and theoretical foundation for evaluating risk and return, these fundamental principles involve allocating assets with the aim of minimizing the standard deviation (or, conversely, the variance) and maximizing the expected return. This process is commonly referred to as mean-variance analysis. Utilizing mathematical theoretical models is the norm, but practical applications in the initial phases become challenging due to real-world limitations.

2.1 Evolution of Portfolio Optimization Models

Over time, MPT has evolved from single-factor models (such as MM) to multi-factor models as a result of various market circumstances and requirements, such as the Capital Asset Pricing Model (CAPM) and the Fama-French three-factor model. In order to enhance the comprehension of asset returns, these models incorporate complementary risk factors, including size and value (Idowu, Strüber & Berger (2021); Fama and French, 1992).

2.2 Critiques and Limitations of MPT

MPT relies heavily on two key assumptions: the rationality of investors and the normal distribution of returns. However, it is important to note that both assumptions may not accurately reflect reality. While the standard deviation of returns represents a valid and widely used proxy for risk, they do not specifically address the potential loss or downside risk of any asset.

Moreover, the standard deviation stays not constant and fluctuates over several time intervals. In the future, the correlations between various assets and asset classes will remain the same. Note that the correlations are also variable and will fluctuate across different time intervals. For cases where these assumptions are invalid, MPT may not be an effective tool. To a certain extent, MPT has limitations in its ability to optimize a portfolio just for idiosyncratic risks, which refer to hazards specific to individual assets. It is unable to optimize for market risks that impact the entire portfolio.

Both Thaler, Richard & Cass (2008) and Banerjee (1992) highlighted the importance of incorporating psychological factors into investment models since behavioral finance theories, such as herding behavior and prospect theory, question the assumption of rational decision-making. Investors tend to engage in untimely buying and selling of stocks without adhering to a specific diversification discipline, thereby reducing their performance relative to the overall market.

2.3 Recent Developments in Portfolio Optimization

The utilization of sophisticated optimization techniques, such as machine learning algorithms and big data analysis, will be facilitated by technological process, improved data accessibility, and the advancement of AI. These methods will be employed to enhance the efficiency and real outcomes of MPT. According to Idowu, Strüber & Berger (2021), these methods have the capability to examine vast datasets and uncover intricate patterns in financial markets, hence facilitating model analysis and keeping them up to date.

Despite the limitations of MPT, it nonetheless serves as a significant framework for understanding and controlling portfolio risk. This study expands upon the works of Yu & Zhang (2023), who stressed the importance of incorporating estimation errors and model uncertainty. They also demonstrated` the effectiveness of MPT under various market conditions. To enhance the risk-adjusted returns of the entire portfolio, MPT greatly amplifies the asset diversification. Diversifying a portfolio by mixing uncorrelated or negatively correlated assets, meaning they tend to move in opposite directions compared to other assets, improves the overall performance of the portfolio. The combination of higher-risk assets, such as equities, with lower-risk assets, such as bonds or cash, generally yields superior long-term risk-return characteristics than either asset class individually. Which means that implement a strategy of diversification across different asset classes and individual assets. Having many stocks helps with diversification, but it is even more crucial to combine stocks with bonds, government securities, cash, gold, or other types of assets.

Furthermore, BKKBN Provinsi Aceh (2018) presents a pertinent case study that illustrating the importance of cultural factors in the process of decision-making. This case study allows for comparison with the impact of real-world constraints on investment strategies.

3. METHOLOGY

3.1 Data Preparation

This study utilizes data based on daily returns over a 20-year timeframe for ten stocks in the technology, financial, and services industries, as well as the S&P 500 Index. To ensure the consistency with the assumption of monthly rebalancing, the study aggregates daily returns on a monthly basis to generate monthly return data for the portfolio optimization model. This step mitigates the effects of short-term market fluctuations in the specific case, maximizes uncertainty in conjunction with monthly return amounts, and guarantees that the analysis aligns with the long-term investment approach typically assumed in Modern Portfolio Theory (MPT). The data was obtained from a Bloomberg picture deck that was provided, which contains details about each stock, their respective industry groups, along with the records of the stock's trailing gains.

Key calculations include:

- Expected Returns: The average monthly returns over the 20-year duration.

- Covariance Matrix: A figure created with Excel's 'COVARIANCE.P' function, which depicts the degree to which the returns of different assets move together.
- Beta Values: Each stock's beta in relation to the S&P 500 is calculated for the Index Model.

3.2 Optimization Process

3.2.1 Markowitz Model (MM)

The objective of the Markowitz Model is to minimize the variability of a portfolio's variance while simultaneously achieving a desired level of return. The optimization problem is delineated by the subsequent equation:

$$\sigma_p^2 = w^{\mathrm{T}} \sum w$$

where w represents the portfolio weights and ∑ the covariance matrix. The constraints include:

- The sum of portfolio weights equals 1.
- Additional limitations unique to a given scenario (Five constraints, see 3.3 Constraints Scenarios).

The portfolio on the efficient frontier is determined for each constraint scenario by utilizing Excel's Solver feature.

3.2.2 Index Model (IM)

By assuming that asset returns are predominantly determined by their relationship to a market index (such as the S&P 500), as indicated by beta values, the Index Model streamlines optimization. We model the variance of the portfolio as follows:

$$\sigma_p^2 = \beta_p^2 \sigma_{SPX}^2 + \sigma_{?p}^2$$

It is worth noting that, according to Yu & Zhang (2023), although this method reduces computational complexity and is beneficial to experimental purposes, it may underestimate the advantages of diversification. In practice, using beta as the sole indicator of an asset's risk may result in suboptimal portfolios, especially in volatile markets where additional risk factors may be present.

Additionally, BKKBN Provinsi Aceh (2018) highlights the importance of cultural factors in decision-making, which can be used as an analogy to the impact of real-world constraints on investment strategies. Although the impact of strategies such as cultural factors on investment theory was not shown in the experiment, it is worth mentioning that by understanding these constraints, investors could become more adept at negotiating the complexities of portfolio optimization.

3.3 Constraint Scenarios

We examine five possibilities with constraints:

- Constraint 1 (Regulation T): Implement leverage limits, which simulate FINRA's Regulation T, involving restrictions on leverage and account equity:

$$\sum_{i=1}^{11} |w_i| \leqslant 2$$

- Constraint 2 (maximum asset weight): The proportion of any asset in the investment portfolio must not exceed 25%. This can be represented as some form of maximum weight restriction:

$$|w_i| \leqslant 1,\ for \forall_i$$

- Constraint 3 (unconstrained): There are no additional optimization constraints, and the composition of the portfolio is unrestricted:

$$NoConstraints$$

- Constrain 4 (No Short Selling): Simulates the restrictions in the U.S. mutual fund industry where no short positions are allowed, and every weight needs to be positive or 0:

$$w_i \geqslant 0,\ for \forall_i$$

- Constrain 5 (Beta-Weighted Portfolio): Incorporate a broad index (such as the S&P 500) into the portfolio to consider its impact on beta risk:

$$w_1 = 0$$

4. RESULTS

4.1 Efficient Frontier and Minimal Risk Portfolio

4.1.1 IM Model Performance

Graph 1: Efficient Frontier for Index Model under Different Constraints

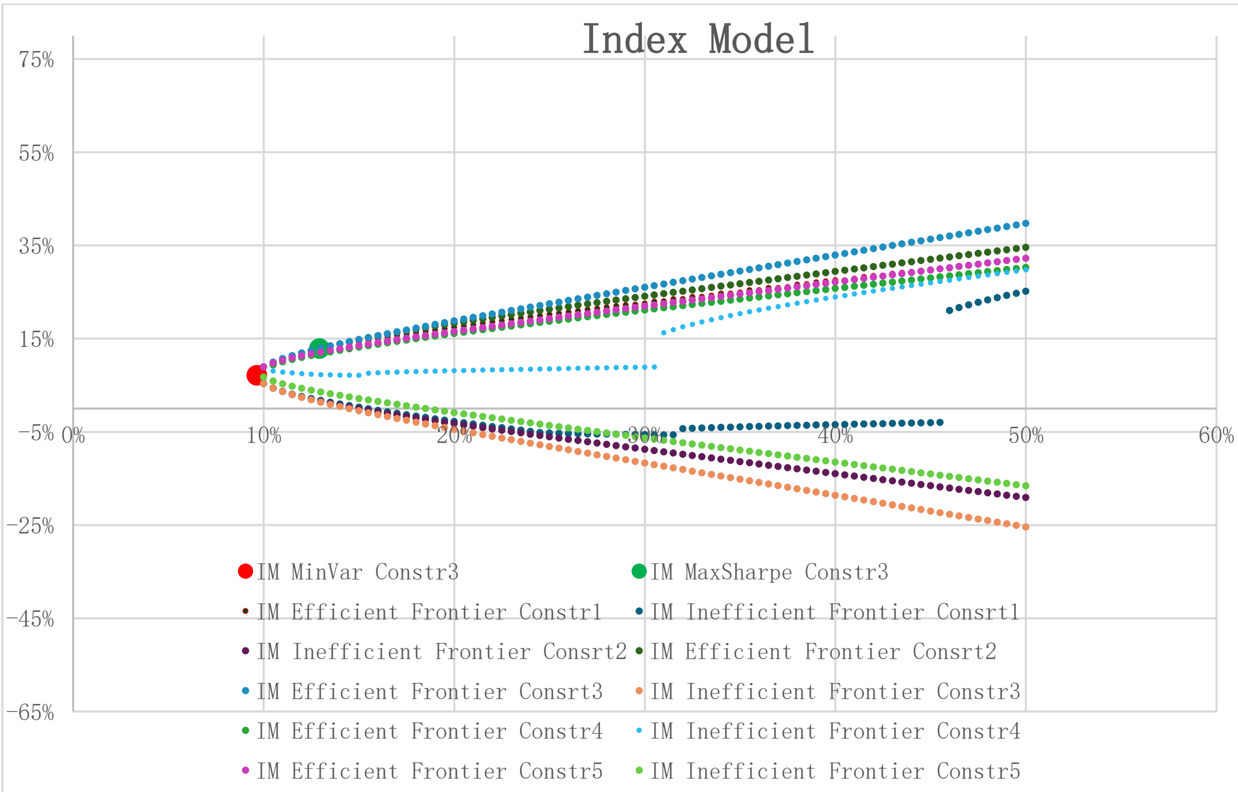


Table 1: Minimal Risk Portfolio Composition under Each Constraint (IM Model)

IM (Constr1):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	25.62%	-4.04%	-5.27%	-2.81%	-8.73%	0.76%	10.28%	-1.40%	31.27%	27.68%	26.64%	7.15%	9.63%	0.742
MaxSharpe	-47.62%	8.88%	-1.24%	-0.49%	-0.64%	6.67%	29.55%	4.57%	43.95%	33.35%	23.01%	12.07%	12.18%	0.990
IM (Constr2):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	25.62%	-4.04%	-5.27%	-2.81%	-8.73%	0.76%	10.28%	-1.40%	31.27%	27.67%	26.64%	7.15%	9.63%	0.742
MaxSharpe	-70.16%	10.32%	-0.57%	-0.11%	0.57%	9.39%	34.25%	7.35%	46.90%	36.94%	25.11%	12.87%	12.92%	0.996
IM (Constr3):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	25.62%	-4.04%	-5.27%	-2.81%	-8.73%	0.76%	10.28%	-1.40%	31.27%	27.67%	26.64%	7.15%	9.63%	0.742
MaxSharpe	-70.16%	10.32%	-0.57%	-0.11%	0.57%	9.39%	34.25%	7.35%	46.90%	36.94%	25.11%	12.87%	12.92%	0.996
IM (Constr4):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.21%	0.00%	33.55%	28.89%	28.35%	8.64%	10.16%	0.850
MaxSharpe	0.00%	6.74%	0.00%	0.00%	0.00%	0.00%	17.75%	0.00%	37.34%	22.75%	15.41%	10.71%	11.72%	0.914
IM (Constr5):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	0.00%	-3.35%	-3.28%	-1.14%	-6.02%	3.27%	14.25%	1.13%	34.13%	31.54%	29.47%	7.82%	9.75%	0.802
MaxSharpe	0.00%	7.68%	-5.84%	-4.45%	-6.81%	2.56%	22.82%	0.47%	38.76%	26.69%	18.11%	10.84%	11.48%	0.944

4.1.2 MM Model Performance

Graph 2: Efficient Frontier for Markowitz Model under Different Constraints

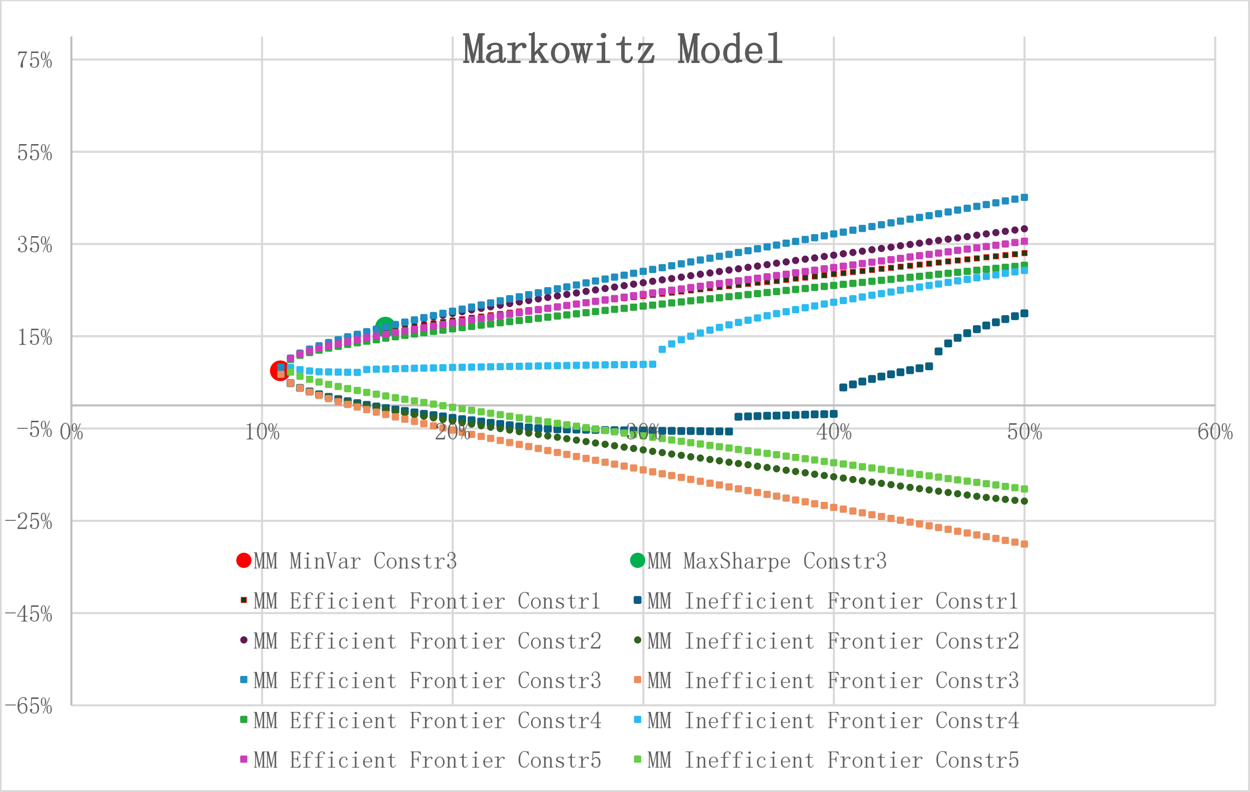


Table 2: Minimal Risk Portfolio Composition under Each Constraint (MM Model)

MM (Constr1):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	38.37%	-2.97%	-2.89%	1.33%	-5.90%	-0.30%	19.41%	-11.48%	25.93%	18.83%	19.67%	7.51%	10.95%	0.685
MaxSharpe	-42.74%	15.75%	-1.15%	-6.11%	3.25%	6.48%	35.29%	1.07%	45.71%	30.00%	12.45%	14.01%	13.95%	1.004
MM (Constr2):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	38.37%	-2.97%	-2.89%	1.33%	-5.90%	-0.30%	19.41%	-11.48%	25.93%	18.83%	19.67%	7.51%	10.95%	0.685
MaxSharpe	-100.00%	21.50%	0.31%	-8.15%	11.46%	12.25%	44.92%	6.87%	52.33%	41.02%	17.48%	16.56%	16.06%	1.031
MM (Constr3):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	38.37%	-2.97%	-2.89%	1.33%	-5.90%	-0.30%	19.41%	-11.48%	25.93%	18.83%	19.67%	7.51%	10.95%	0.685
MaxSharpe	-109.97%	22.46%	0.89%	-8.19%	12.73%	13.21%	46.46%	7.90%	53.50%	42.72%	18.30%	16.99%	16.48%	1.031
MM (Constr4):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	9.49%	0.00%	0.00%	0.00%	0.00%	0.00%	19.85%	0.00%	28.91%	20.62%	21.13%	8.88%	11.27%	0.788
MaxSharpe	0.00%	10.95%	0.00%	0.00%	0.00%	0.00%	23.73%	0.00%	42.56%	16.17%	6.60%	12.06%	13.12%	0.919
MM (Constr5):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	0.00%	-0.97%	0.08%	2.51%	-0.99%	3.50%	24.70%	-8.17%	28.91%	25.58%	24.85%	8.71%	11.18%	0.779
MaxSharpe	0.00%	14.93%	-6.85%	-10.14%	-1.30%	2.43%	30.65%	-2.27%	43.31%	23.61%	5.64%	13.06%	13.69%	0.954

4.1.3 Explanation of Efficient Frontier trends and Minimal Risk Portfolio

The Efficient Frontier and Inefficient Frontier are plotted for each scenario shown in graph 1 and graph 2, respectively. This allows us to identify the portfolio that provides the maximum expected return at a given amount of risk level. The efficient frontier depicts the highest achievable return under a given risk level, or the lowest feasible risk display under a given return ratio. In a graph, an efficient frontier is usually presented as a curve divided according to the likelihood under each constraint, with upper points having higher Sharpe ratios than lower points. The points on the inefficient frontier do not achieve the optimal risk-return trade-off on the efficient frontier, so they are usually not adopted by investors. As shown in the graph, these points are usually located below or inside the efficient frontier (surrounded by arc lines).

In addition, the portfolio with the minimal risk is determined by analyzing the graphs. It is observed that this portfolio is located at the lowest point of the efficient frontier, as depicted in table 1 and table 2.

From the perspective of the different curves presented by the five constraints, regardless of whether it is IM Model or MM Model, the efficient frontier and inefficient frontier at the highest and lowest points occur under the same constraints. Constraint 3 (unrestricted) shows the efficient frontier curve at the highest position (0.397 and 0.450 at the 50% point). On the other hand, the efficient frontier at the lowest position is usually Constraint 4 (no short selling, that is, each weight is a non-negative number) (approximately 0.302 and 0.303 at the 50% point). Constraint 4 also represents the lowest point of the inefficient frontier in both images (0.298 and 0.292 at the 50% point), while conversely the lowest points in the inefficient frontier usually occur at Constraint 3 (-0.254 and -0.301 at the 50% point).

4.2 Maximal Sharpe Ratio Portfolio

4.2.1 IM Model Performance

Graph 3: Maximal Sharpe Ratio Portfolio (IM Model)

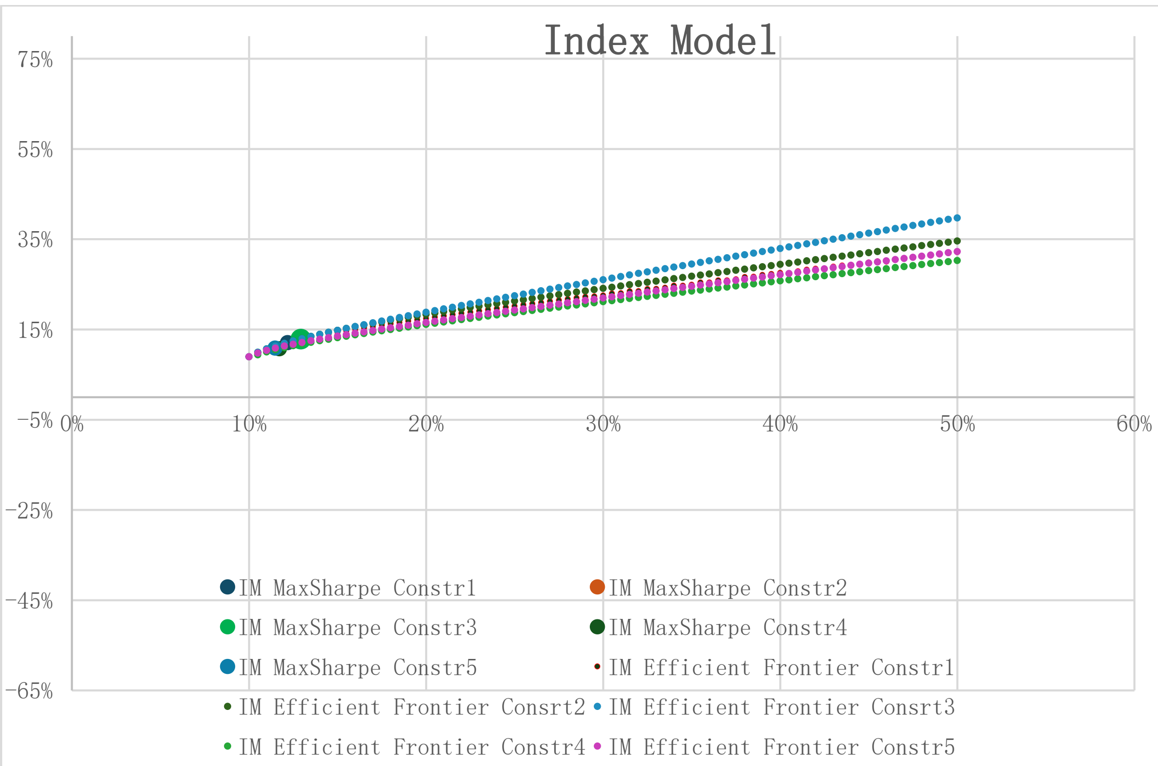


Table 3: Portfolio Weights and Risk Metrics for Maximal Sharpe Ratio Portfolio (IM Model)

IM (Constr1):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	25.62%	-4.04%	-5.27%	-2.81%	-8.73%	0.76%	10.28%	-1.40%	31.27%	27.68%	26.64%	7.15%	9.63%	0.742
MaxSharpe	-47.62%	8.88%	-1.24%	-0.49%	-0.64%	6.67%	29.55%	4.57%	43.95%	33.35%	23.01%	12.07%	12.18%	0.990
IM (Constr2):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	25.62%	-4.04%	-5.27%	-2.81%	-8.73%	0.76%	10.28%	-1.40%	31.27%	27.67%	26.64%	7.15%	9.63%	0.742
MaxSharpe	-70.16%	10.32%	-0.57%	-0.11%	0.57%	9.39%	34.25%	7.35%	46.90%	36.94%	25.11%	12.87%	12.92%	0.996
IM (Constr3):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	25.62%	-4.04%	-5.27%	-2.81%	-8.73%	0.76%	10.28%	-1.40%	31.27%	27.67%	26.64%	7.15%	9.63%	0.742
MaxSharpe	-70.16%	10.32%	-0.57%	-0.11%	0.57%	9.39%	34.25%	7.35%	46.90%	36.94%	25.11%	12.87%	12.92%	0.996
IM (Constr4):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.21%	0.00%	33.55%	28.89%	28.35%	8.64%	10.16%	0.850
MaxSharpe	0.00%	6.74%	0.00%	0.00%	0.00%	0.00%	17.75%	0.00%	37.34%	22.75%	15.41%	10.71%	11.72%	0.914
IM (Constr5):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	0.00%	-3.35%	-3.28%	-1.14%	-6.02%	3.27%	14.25%	1.13%	34.13%	31.54%	29.47%	7.82%	9.75%	0.802
MaxSharpe	0.00%	7.68%	-5.84%	-4.45%	-6.81%	2.56%	22.82%	0.47%	38.76%	26.69%	18.11%	10.84%	11.48%	0.944

Total	MM(Constr1)	MM(Constr2)	MM(Constr3)	MM(Constr4)	MM(Constr5)
MinVar	=SUM(P3:Z3)	100.00%	100.00%	100.00%	100.00%
Formula	S	SUM(number1, [number2], ...)	P9:Z9)	SUM(P12:Z12)	SUM(P15:Z15)
MaxSharpe	100.00%	100.00%	100.00%	100.00%	100.00%
Formula	SUM(P4:Z4)	SUM(P7:Z7)	SUM(P10:Z10)	SUM(P13:Z13)	SUM(P16:Z16)

4.2.2 MM Model Performance

Graph 4: Maximal Sharpe Ratio Portfolio (MM Model)

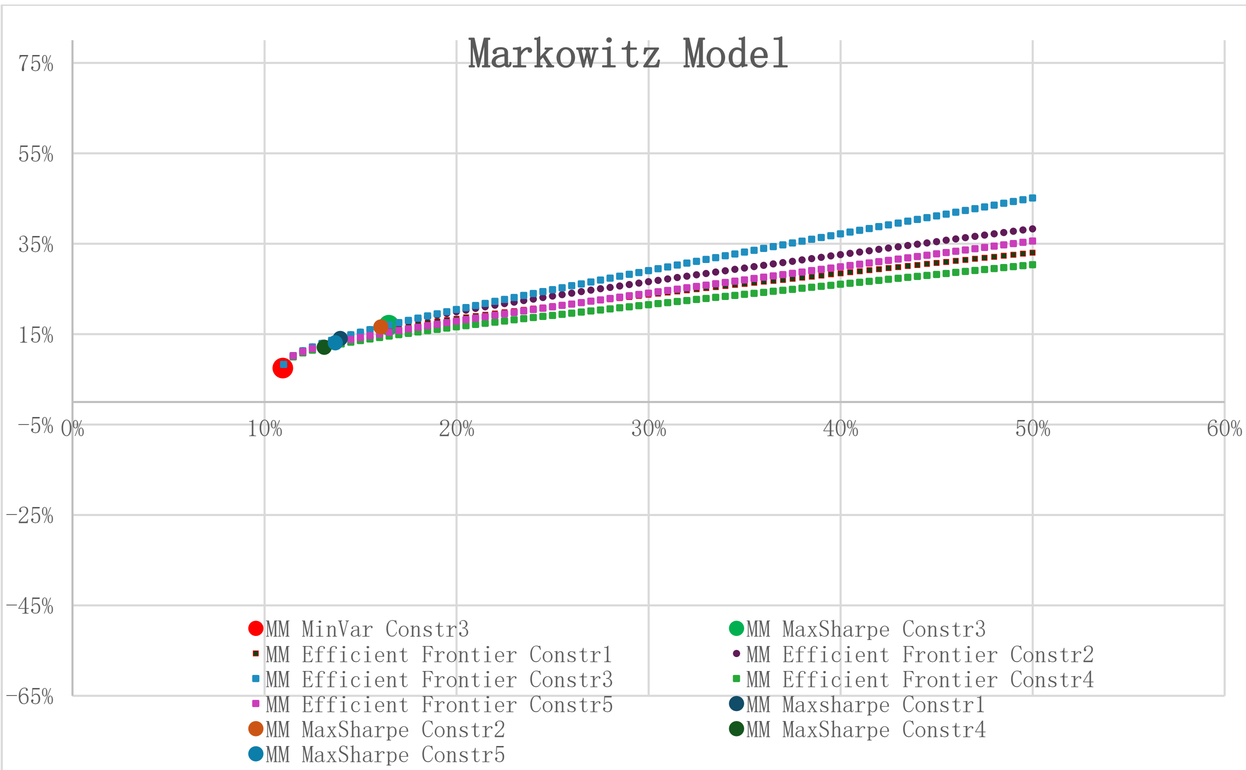


Table 4: Portfolio Weights and Risk Metrics for Maximal Sharpe Ratio Portfolio (MM Model)

MM (Constr1):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	38.37%	-2.97%	-2.89%	1.33%	-5.90%	-0.30%	19.41%	-11.48%	25.93%	18.83%	19.67%	7.51%	10.95%	0.685
MaxSharpe	-42.74%	15.75%	-1.15%	-6.11%	3.25%	6.48%	35.29%	1.07%	45.71%	30.00%	12.45%	14.01%	13.95%	1.004
MM (Constr2):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	38.37%	-2.97%	-2.89%	1.33%	-5.90%	-0.30%	19.41%	-11.48%	25.93%	18.83%	19.67%	7.51%	10.95%	0.685
MaxSharpe	-100.00%	21.50%	0.31%	-8.15%	11.46%	12.25%	44.92%	6.87%	52.33%	41.02%	17.48%	16.56%	16.06%	1.031
MM (Constr3):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	38.37%	-2.97%	-2.89%	1.33%	-5.90%	-0.30%	19.41%	-11.48%	25.93%	18.83%	19.67%	7.51%	10.95%	0.685
MaxSharpe	-109.97%	22.46%	0.89%	-8.19%	12.73%	13.21%	46.46%	7.90%	53.50%	42.72%	18.30%	16.99%	16.48%	1.031
MM (Constr4):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	9.49%	0.00%	0.00%	0.00%	0.00%	0.00%	19.85%	0.00%	28.91%	20.62%	21.13%	8.88%	11.27%	0.788
MaxSharpe	0.00%	10.95%	0.00%	0.00%	0.00%	0.00%	23.73%	0.00%	42.56%	16.17%	6.60%	12.06%	13.12%	0.919
MM (Constr5):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	0.00%	-0.97%	0.08%	2.51%	-0.99%	3.50%	24.70%	-8.17%	28.91%	25.58%	24.85%	8.71%	11.18%	0.779
MaxSharpe	0.00%	14.93%	-6.85%	-10.14%	-1.30%	2.43%	30.65%	-2.27%	43.31%	23.61%	5.64%	13.06%	13.69%	0.954

Total	IM(Constr1)	IM(Constr2)	IM(Constr3)	IM(Constr4)	IM(Constr5)
MinVar	100.00%	=SUM(AI6:AS6)	100.00%	100.00%	100.00%
Formula	SUM(AI3:AS3)	SUM(AI6:AS6)	SUM(AI9:AS9)	SUM(AI12:AS12)	SUM(AI15:AS15)
MaxSharpe	100.00%	100.00%	100.00%	100.00%	100.00%
Formula	SUM(AI4:AS4)	SUM(AI7:AS7)	SUM(AI10:AS10)	SUM(AI13:AS13)	SUM(AI16:AS16)

4.2.3 Results from Analyzing Maximal Sharpe Ratio

The Sharpe ratio, a metric that quantifies the return on investment adjusted for risk, was optimized for each scenario. According to the information presented in Graph 3 and Graph 4, it can be observed that the Maximum Sharpe

Ratio of IM Model is relatively concentrated under five constraints conditions. This reflects that IM Model under different risk levels remains consistent with the index across various degrees of risk. Rather than pursuing the optimal risk-return ratio, it is therefore less susceptible to the risk. The Maximum Sharpe Ratio of MM Model exhibited a wider range and even a higher value under different constraints.

Although this is uncertain in a single stock, the collective outcomes of ten stocks consistently demonstrate this pattern. The reason for this is that the Markowitz model provides a more precise estimation of risk than the exponential model, therefore resulting in a generally higher Sharpe ratio. Table 3 and Table 4 mainly illustrate the specific value of Maximum Sharpe Ratio under various constraints. The ultimate result is located on the far right of the table, and the accuracy is determined by verifying that the sum of the respective values of ten stocks amounts is 100%.

4.3 Capital Allocation Line (CAL) and Minimal Return Frontier

4.3.1 IM Model Performance

Graph 5: Capital Allocation Line (CAL) For IM Model

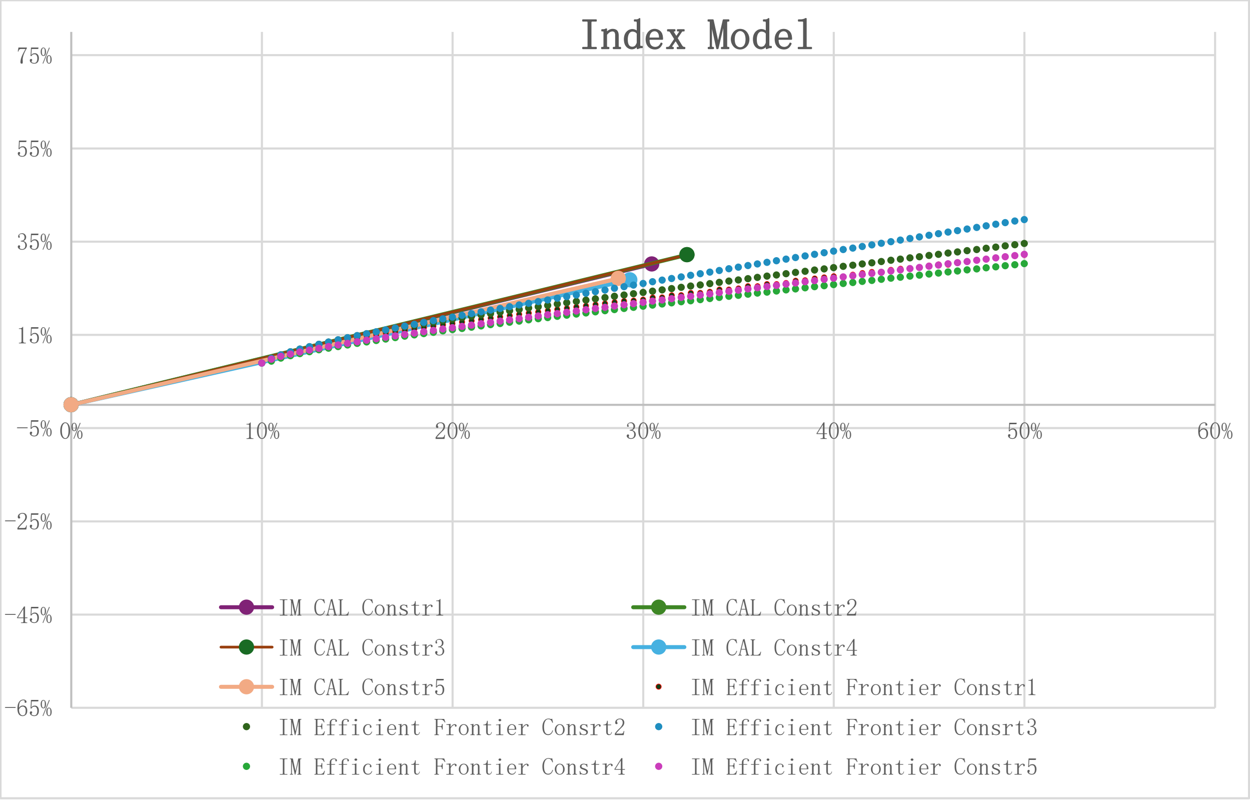
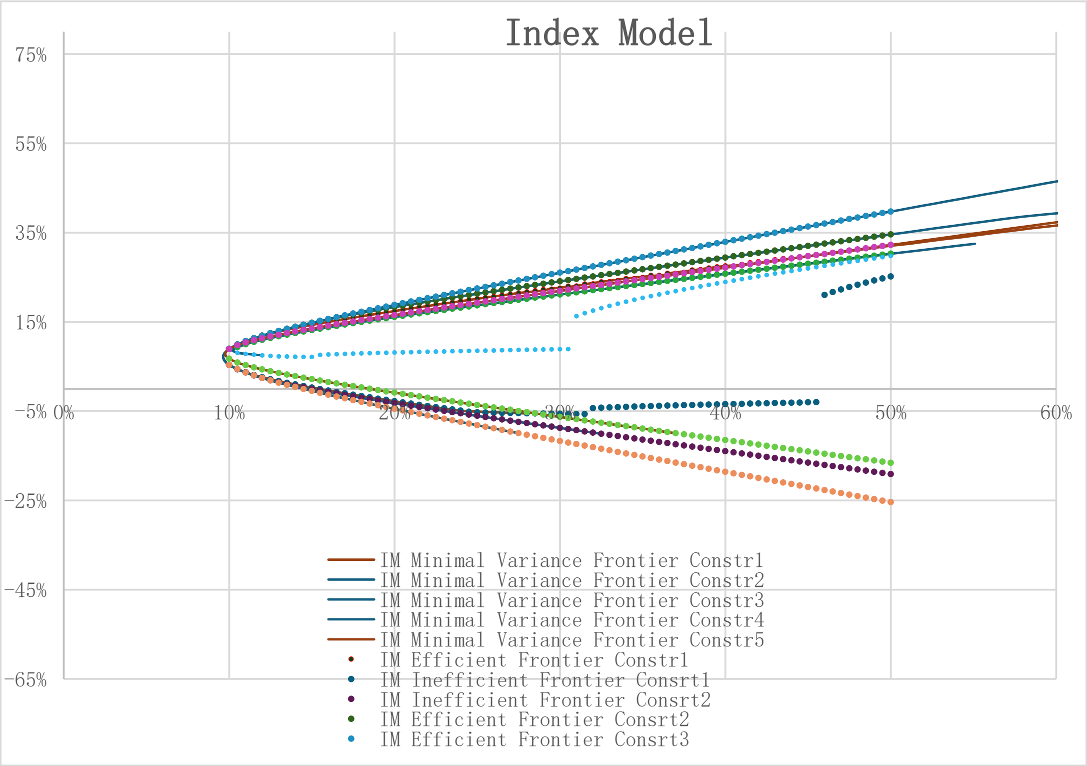


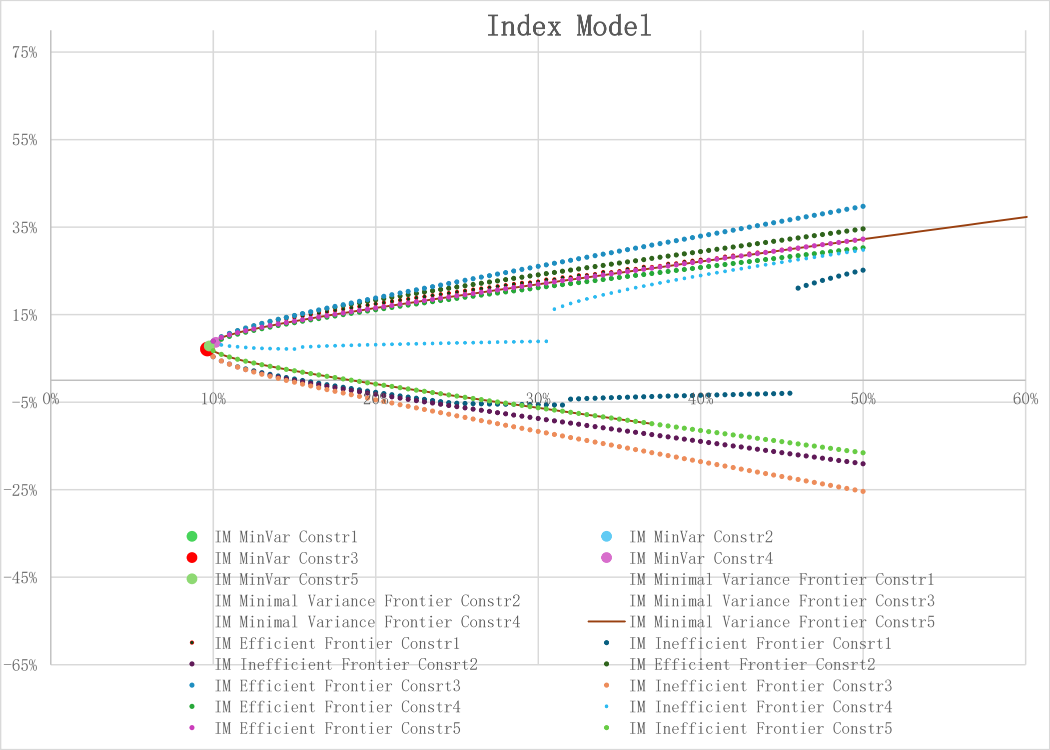
Table 5: CAL Data for IM Model

CAL:	0.0%	0.0%
2.5	30.2%	30.5%
CAL:	0.0%	0.0%
2.5	32.2%	32.3%
CAL:	0.0%	0.0%
2.5	32.2%	32.3%
CAL:	0.0%	0.0%
2.5	26.8%	29.3%
CAL:	0.0%	0.0%
2.5	27.1%	28.7%

Graph 6: Minimal Return Frontier under Different Constraints (IM Model)



Graph 7: Minimal Variance for IM Model



4.3.2 MM Model Performance

Graph 8: Capital Allocation Line (CAL) For MM Model

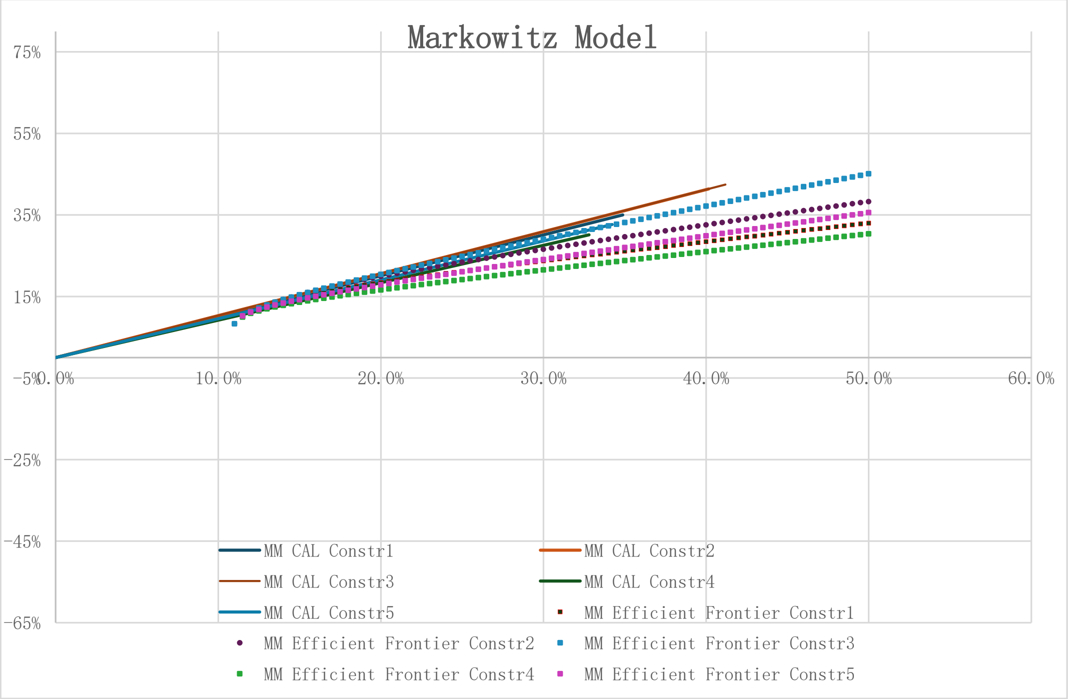
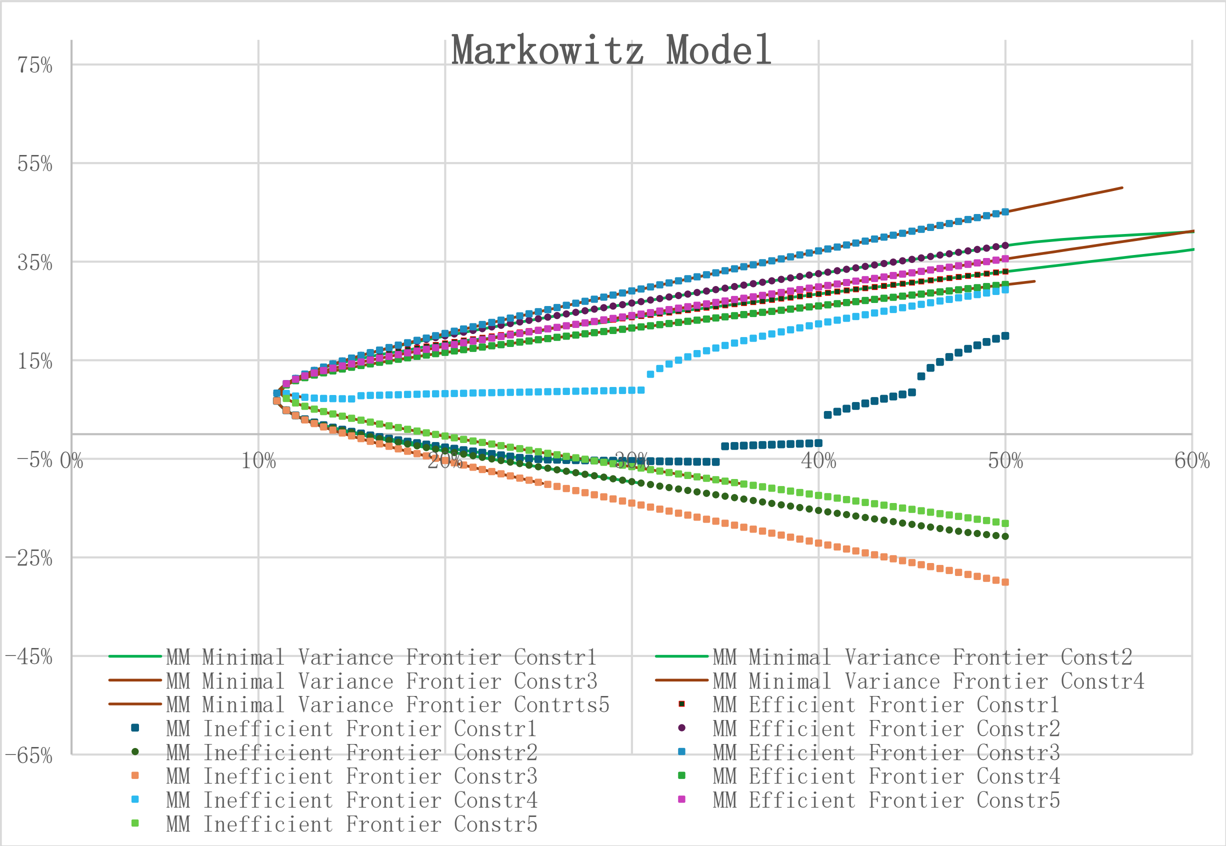


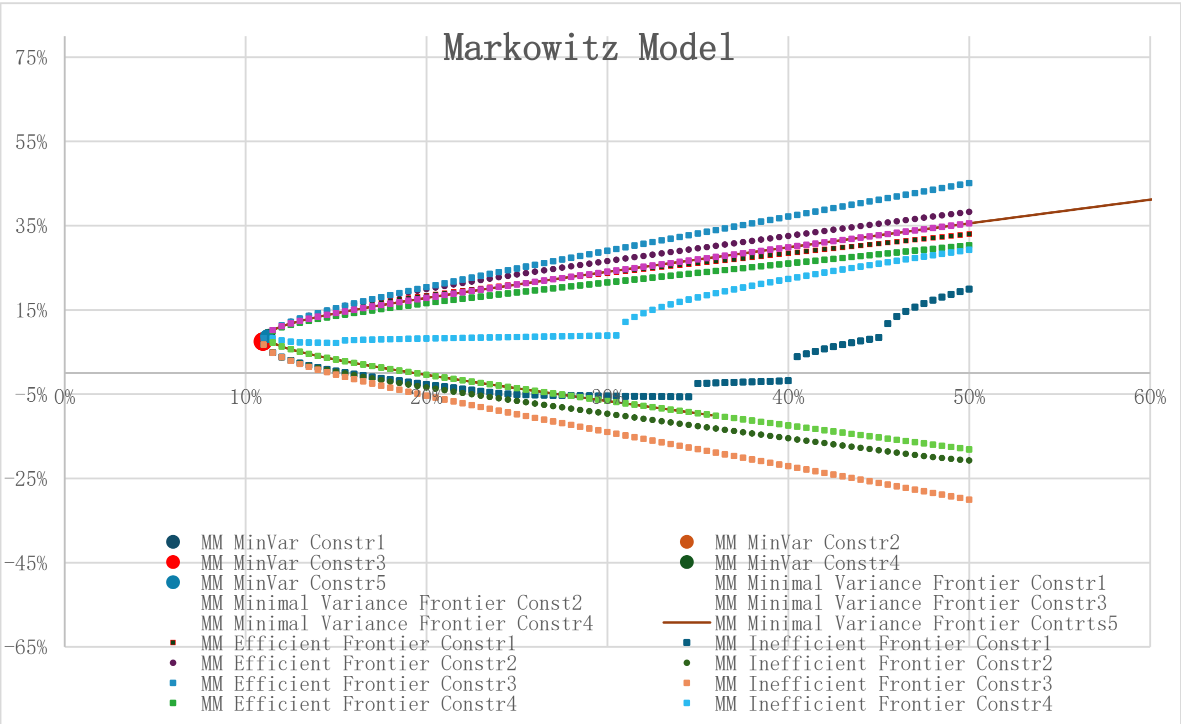
Table 6: CAL Data for MM Model

CAL:	0.0%	0.0%
2.5	35.0%	34.9%
CAL:	0.0%	0.0%
2.5	41.4%	40.2%
CAL:	0.0%	0.0%
2.5	42.5%	41.2%
CAL:	0.0%	0.0%
2.5	30.1%	32.8%
CAL:	0.0%	0.0%
2.5	32.6%	34.2%

Graph 9: Minimal Return Frontier under Different Constraints (MM Model)



Graph 10: Minimal Variance for MM Model



4.3.3 Results from CAL by Comparation

The primary objective of the research shown in the graphs and tables above is to include risk-free assets into the portfolio and to derive a trade-off between risk and return, the effect of balance is illustrated by a plotted capital distribution line (CAL) as seen in Graph 5 and Graph 8. Markowitz's model has exceptional performance, as evidenced by the more pronounced incline of the CAL, which is partly associated with the Sharpe Ratio. The slope data can be obtained from Table 5 and Table 6. The CAL slope of the MM Model is always greater than that of the IM Model under different constraints.

4.3.4 Results from Minimal Return Frontier

Furthermore, the study also examines the minimum return frontier, which shows the minimum return that is feasible given certain constraints. The trend function of Minimal return frontier of IM and MM Model presented in the image can be observed in Graph 6 and Graph 9. On this basis, Graph 7 and Graph 10 both incorporate the minimal variance, which represents the minimum return point, in the five distinct constraint scenarios. From the results, the minimal return frontier function graphs presented by IM Model and MM Model basically coincided with their respective efficient frontier, subject to varying constraints.

It is worth mentioning that the analysis from the perspective of minimum return point is consistent with the result presented by Maximum Sharpe Ratio. Under different constraints, minimal variance point of IM Model is relatively dispersed compared with MM Model. In addition, MM Model typically generates a higher minimal return (the specific value under this result can be obtained by using the value of return and standard deviation on the far right of reality in table 1 and table 2). This is because the MM Model allows investors to respond to the lowest possible return point by adjusting their asset allocation and portfolio, thus adopting a defensive strategy when the market is downturn. In contrast, the IM Model is less versatile and adaptable in this regard as its objective is to replicate the index rather than actively respond to changes in the market. In risk management, specific performance usually relies on the selection of indexes or derivatives with low volatility to hedge risks.

In addition, in terms of perspective of constraints, the peak of the Minimal Return Frontier usually occurs in Constrain 3, which is characterized by being unrestricted. The lowest point lies in the premise of Constrain 4, that is, when the ownership weight is greater than or equal to 0. This condition is directly shown by the efficient frontier, providing further evidence of the correlation between this data and the efficient frontier in the context of portfolio market and risks.

4.4 Conclusion Over Results

The analysis demonstrates that since Index Model usually refers to a passive investment strategy aimed at replicating or tracking the performance of a certain market index, its lack of personalized flexibility, which is the biggest difference from Markowitz Model, may lead to reduced diversification of investment portfolios and increased risks.

In this case, the efficient frontier of Markowitz model has stronger adaptability to all kinds of constraints, which is reflected in its relatively concentrated minimal return, steeper capital allocation line (CAL) slope and higher Maximum Sharpe Ratio.

The Markowitz Model enables investors to construct portfolios that minimize risk and maximize return by considering the correlations between assets. The primary feature of this tool is its utilization of mean-variance optimization to identify the most efficient portfolio, which is usually absent in the Index Model. The results of Yu & Zhang (2023) align with this statement, as their research emphasizes the importance of considering model uncertainty and estimation errors in portfolio optimization.

5. DISCUSSION

These findings indicate that it is important to account for the constraints from real-world financial investment markets when initially constructing a portfolio. Since the underlying theory of the MM Model incorporates the complete covariance matrix, compared with the IM Model, it could be able to properly deal with the investment ratio under different constraints, resulting in a more resilient and diversified portfolio. However, despite the high computational efficiency of the Index Model, thanks to its definition of a single beta value, that is, the market tracking index, the relatively simplified steps of the index model may lead to sub-optimal investment portfolios under certain circumstances, because the financial investment market is constantly changing, and many factors need to be weighed and considered. This has prompted investors to recognize the need to carefully evaluate the constraints they encounter, and the trade-offs associated with various optimization strategies.

This study also highlights the significance of comprehending and adjusting to the complexity of the investing environment. It also reinforces the influence of culture on decision making, as articulated in BKKBN Provinsi Aceh (2018). Effective boundaries and ideal portfolio composition are altered by practical constraints, such as the five constraints used in the study above. Therefore, a complex approach to portfolio development requires financial market dynamics as well as objective factors associated with constraints.

5.1 Risk Measures Beyond Robust Calculation and Variance

However, compared with the simplicity of the Index Model, the solution efficiency of the Markowitz Model in application has certain defects. As mentioned above, the calculation amount of a huge portfolio is always a difficult problem. At present, the mainstream solution is to simplify the calculation through a specific algorithm. The essence of mean-variance model is quadratic programming problem. In the case of shorting, the model can be solved by Lagrange multiplier method. On the other hand, in the case of non-shorting, the model analysis is complicated, and the final solution cannot be obtained. Therefore, the academic community generally adopts Monte Carlo, branch and bound, iteration and other methods to solve optimization problems, just like a square by constantly cutting corners to achieve the process of approximating the circle. There are some heuristic algorithms to find the approximate optimal solution of the model, such as the rotation algorithm of inequality group, genetic algorithm and so on.

In addition, to some extent, the use of variance as the only risk indicator in the Markowitz Model has its limitations. The premise assumption of the model is too idealized, and the asymmetry of return, that is, negative return may have a huge impact on portfolio performance, while the variance is not taken into account, so its practicality in the real environment will be limited. Moreover, differences are premised on the assumption of a normal distribution of returns, which may not be accurate in practice, especially during periods of market volatility. Investigating alternative risk metrics, such as conditional value at risk (CVaR), value at risk (VaR), and adverse risk, provides a deeper understanding of portfolio risk. These measures are essential for risk management, which focuses on tail risks and risks that can cause significant losses (Jorion, 2007). However, some scholars believe that strategies such as contrast equal weight, market capitalization weighting and mean-contrast strategy have the best performance and strong applicability in the stock market. Both theories have an experimental basis, and the specific application efficiency is related to different investment markets and even different countries' financial policies, so there is no authoritative unified explanation.

5.2 Parameter Estimation and Model Risk

Financial markets are characterized by inherent uncertainty, which makes it difficult to estimate parameters such as expected returns and covariance matrices. In addition, due to the lack of stability of the Model itself, although the Markowitz Model is less responsive to the estimation error and the influence of the market than the Index Model, the sensitivity of parameters and the estimation error still cannot be completely avoided. As a result, portfolio optimization may be significantly affected by the estimation error, which may lead to poor portfolio allocation. In addition, the choice of optimized models introduces model risk, as different models may produce different results.

According to Rockaflar and Uryasev (2000), portfolio construction can be made more reliable and resilient by studying robust optimization techniques that consider parametric uncertainty and model risk. The robust technique is also a mathematical optimization method that can be viewed as an extension and complement to the Markowitz Model to improve portfolio selection and adjustment by introducing uncertainty sets and worst-case analysis. In robust optimization, the uncertainty parameters are confined to a known set, called the uncertainty set. This set contains all possible uncertainty parameter values. Since its essence is to minimize the potential loss in the worst case, that is, to ensure that the solution is still valid when the deterministic parameter takes the most unfavorable value, the objective function and the constraints are adjusted to account for uncertainty.

5.3 Transaction Costs and Liquidity Constraints

Liquidity constraints and transaction costs are critical practical considerations in portfolio optimization. Liquidity constraint refers to the difficulty of buying and selling assets. Usually, non-current assets may take a long time to be bought and sold at a reasonable price and may face a large price shock. Transaction costs include the costs incurred when buying or selling an asset, such as commissions, stamp duty, slippage, etc. Illiquid assets and high transaction costs can complicate portfolio rebalancing, which can erode investment returns. By incorporating liquidity constraints and transaction costs into the optimization process, more practical and functional portfolio solutions can be achieved (Chow, Kose, & Li, 2016). When the transaction scale is large, investors need to predict the future stock price trend, and consider the cost of large trading orders to impact the market and establish a strategy model to control the minimum transaction cost. For example, additional constraints and objective functions are introduced into portfolio optimization models to account for liquidity constraints and transaction costs, or liquidity constraints can be addressed by adjusting asset weights, introducing liquidity premiums, or limiting transaction frequency.

In addition, the optimization process can include strategies to minimize transaction costs, such as reducing transaction frequency, trading in bulk, or selecting assets with lower transaction costs. This helps ensure that the investment strategy not only works in theory, but also produces good risk-adjusted returns in practice.

5.4 The Role of Technology in Portfolio Optimization

Due to technological advances, there has been a major shift in the field of portfolio optimization. Richer data and higher computing power enable the analysis of larger data sets and the implementation of more complex optimization models. Investors have access to powerful analytics tools and algorithms through software tools and platforms dedicated to portfolio optimization. In addition, quantitative strategies and algorithmic trading are becoming more common, enabling automated and systematic portfolio management (Investopedia, 2024). For example, a machine learning model outperforms a market benchmark on metrics such as the Sharpe ratio, which in this case means that the model builds a portfolio that can earn more than the market average for taking the same or less risk than the market average for the same return.

This is due to the ability of machine learning models to simultaneously process and analyze larger amounts of data, identify complex patterns and investment relationships, and most importantly, automatically adjust portfolios to control risk, while ensuring the stability and reliability of their performance in different market environments, which can help investors adapt more quickly to changing trends in financial markets.

6. CONCLUSION

This paper makes an in-depth analysis of the differences between Markowitz Model and Index model in portfolio optimization, expounds the conclusion that Markowitz Model is superior to Index model in portfolio calculation, and supplements the advantages and disadvantages of both in practical application. The importance of different constraints in practice is emphasized.

The significant advantage of Markowitz Model in practical application is that it can adjust the portfolio structure independently and flexibly according to investors' risk preference and investment objectives, to achieve the optimal risk-adjusted return. At the same time, MM Model can also better cope with market uncertainties and changes through continuous adjustment. However, the Markowitz Model also has some disadvantages, such as high computational complexity, the need for large amounts of data to support, and unavoidable errors.

In contrast, the advantage of the Index Model is that it is simple, does not require a lot of data and computing resources, and can quickly replicate the performance of the market index. But as a result, the Index Model may not be able to fully replicate the performance of the market index and unable to adapt to market uncertainties and changes. In addition, the performance of the Index model under relatively stable beta values is generally not much different from that of the Markowitz model, but the latter has a more comprehensive approach to risk assessment, especially when different constraints are applied.

As the findings highlight, before anticipating a portfolio, investors must carefully assess the constraints they will encounter, including their own risk appetite and investment objectives, changes in market conditions, and trade-offs associated with various optimization strategies.

In addition, the paper expounds and looks forward to the optimization of Markowitz Model and the overall development of Modern Portfolio Theory. It is worth pondering that with the continuous innovation of AI technology and the constant changes of the market, Modern Portfolio Theory is also constantly developing and improving. The future portfolio optimization model may pay more attention to the balance of risk and return, pay more attention to the dynamic changes of the market, and even pay more attention to the individuation and customization of investors.

7. FUTURE RESEARCH

In future research, the following can be used as several potential directions for improvement:

7.1 Application to Different Asset Classes

Like the Markovitz Model and Index Model for the stock market, these model analyses can be extended to other asset classes including bonds and real estate in the future. This helps to evaluate the dynamics and sensitivity of different asset classes under market fluctuations, as well as the degree of investment return of stocks corresponding to different asset classes in Modern Portfolio Theory, and further extend to their complementarity in portfolio.

7.2 Dynamic Portfolio Optimization

By Rockafellar and Uryasev (2000), adjusting portfolios to market changes over time, that is, by studying dynamic rebalancing strategies, can strengthen the practical implementation of these models to help investors better cope with market fluctuations, which can provide valuable insights into the evolution of portfolios over time and long-term prospects.

7.3 Incorporation of Behavioral Factors

As proposed by Yu & Zhang (2023), the predictive potential and relevance of portfolio optimization models can be improved by incorporating behavioral factors into portfolio optimization models. The research of behavioral finance shows that market sentiment and investor behavior logic have an important impact on investment decisions (especially individual investors).

7.4 Robust Optimization Techniques

In the case of large market fluctuations, robust optimization technology can improve the adaptability of the portfolio to the market, that is, optimize its elasticity. As mentioned earlier, this technique can reduce the impact of estimation errors and model uncertainty on investment decisions, further improving portfolio stability. (Idowu, Strüber & Berger, 2021)

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