Dean&Francis

ISSN 2959-6130

The role of mathematics in economics: From theoretical deepening to empirical analysis

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Abstract:

This paper discusses the application of mathematics in economics and its importance to the subject's progress. With the increasing application of simplified mathematical methods in economic research and the rising trend of empirical research, the degree of dependence of economics on mathematical analysis has decreased. However, relying on advanced mathematical tools to comprehensively understand and accurately model complex economic phenomena. This tendency to "de-mathematize" can lead to incomplete or misleading conclusions that undermine the credibility and effectiveness of the field. The article points out that the relationship between mathematics and economic theory is complementary rather than antagonistic. Complex theories must be translated into practical insights to bridge the gap between academic discussion and policy practice. Therefore, economists should devote themselves to using mathematical tools to promote the development of economic thought. By improving their mathematical skills, economists can gain a deeper understanding of economic theory, enhance its explanatory and predictive power, and make economics a more robust scientific system. In the future, the application of mathematics in economics will not only be a tool for academic research but also an important force to promote the development of economic theory and practical policy making. This paper further explores the future application prospects of mathematics in economics and its challenges in providing new ideas and directions for economic researchers.

Keywords: Mathematical tools; Economic research; Empirical analysis; Demathematics; Black-Scholes models

1. Introduction

It is becoming increasingly common for economics to employ mathematics in its analysis. Using mathematical tools to quantify economic indicators, combining economic theory with statistical analysis provides greater precision and effectiveness in determining policy changes. It appears that mathematics can contribute to the scientific and rigorous nature of economics, as it can enhance the scientific and rigorous nature of the discipline. For example, mathematics has been a major area of research for many Nobel Prize winners in economics (Lindbeck, 1985). Mathematical models have become indispensable tools for understanding and predicting economic phenomena, from macro to microeconomics, financial engineering to behavioural economics.

Economists have emphasized empirical analysis utilizing causal regression models, such as difference-in-differences and synthetic control methods, interacting with the machine learning approach since the advent of computational capabilities has led to an "empirical revolution" since the early 2000s. At present, many empirical studies use simple regression analysis in economics. These studies use data and statistical techniques to verify hypotheses and provide strong evidence to support or refute specific economic theories. However, this trend also raises the question of whether an over-reliance on simple mathematical tools might limit the development of economics.

Although much empirical research uses less mathematical analysis, in-depth understanding and accurate modelling of complex economic phenomena often require advanced mathematical tools. Thus, economics may be limited in depth and rigour by its reliance on relatively simple mathematics. Demathematizing can lead to incomplete or misleading conclusions that damage the field's credibility and reduce and reduce its effectiveness.

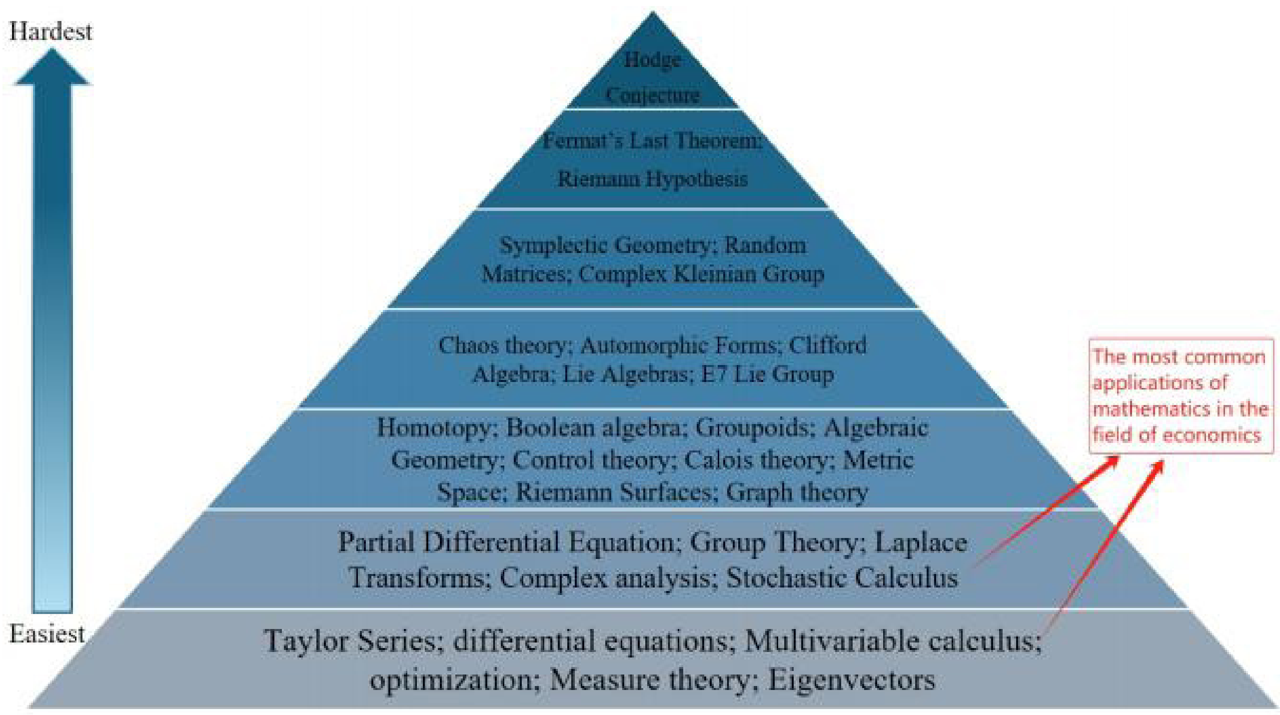

Figure 1: Levels of difficulties in mathematics and Economics (based on my surveys and interviews with math and economics professors)

2. Analysis of the reasons why there is no excessive mathematics in economics

2.1 The mathematical tools are relatively simple

First, the mathematical methods used in economic theory are often more basic and intuitive than those used in purely mathematical fields of study. According to the survey and interview results of professors in mathematics and economics (as shown in Figure 1), linear algebra, calculus, and optimization methods are the most common mathematical methods economists use in constructing theoretical models. Although these techniques may seem complex to the non-expert, they are relatively elementary in the whole system of mathematics. The way these tools are applied tends to focus on solving real economic problems rather than delving into the abstract mathematical principles behind them. For example, when dealing with issues such as market equilibrium or cost minimization, economists are primarily concerned with using existing mathematical knowledge to build models and derive results that help to understand reality. When some people find economics challenging to understand, the problem may have more to do with the limitations of an individual's mathematical background than with economics's reliance on mathematical applications. It is often easier to describe a concept in economics using simple graphics or examples than going for complex mathematical derivation. Moreover, due to the advancement in the area of educational technology, there are more easily accessible and diversified materials and tools that can assist students enhance their mathematical literacy and, therefore, be in a better position to deal with the challenges that are associated with economics courses.

2.2 The Field of Economics Is Not Heavily Mathematical at Present

In particular, the number of empirical analysis papers published in peer-reviewed economics journals since 2000 has grown considerably (see Figure 2). It implies a growing need to use data to test theoretical hypotheses or policy impacts instead of mathematical models. This trend aligns with the recent development in modern economic research, where less focus is placed on the models' aesthetics or complexity. It is, therefore, possible to argue that mathematical and empirical analysis have revolutionized economic theory since most important economic discoveries are made through statistical analysis.

The qualitative study of economics has gradually transformed into the quantitative research of economics over the years, and it is now moving progressively towards the scientific approach. In recent years, a significant focus has been placed on scientific literacy through quantitative methods in economic education, as illustrated by Wilkinson (1999).

Mathematics is becoming essential for cutting-edge research in many disciplines. The development of mathematical science enhances the precision and breadth of research across a wide range of disciplines and promotes interdisciplinary collaboration (see Table 1). It reflects the increasing importance of quantitative analysis in scientific research and the potential of scientific methods in revealing and analyzing economic phenomena.

Notably, the proportion of empirical papers that use little mathematics in top and general economics journals has increased significantly, indicating a shift from mathematical modelling to empirical analysis due to technological advances and the ease of use of analytical software such as Stata, R, and Python. In turn, this software has contributed to the development of empirical research. Both Table 1 and Figure 1 indicate that the increasing number of such empirical papers published in economic journals suggests that causal inference and regression are becoming increasingly popular within various economic subfields in which relatively simple mathematics is required. There has been an increase in empirical economics papers over the years. In the meantime, the proportion of mathematical economic theory papers is declining. It is highlighted in Angrist and Krueger (2001) that contemporary economic analysis methods offer deeper insights into economic matters despite not being highly mathematical.

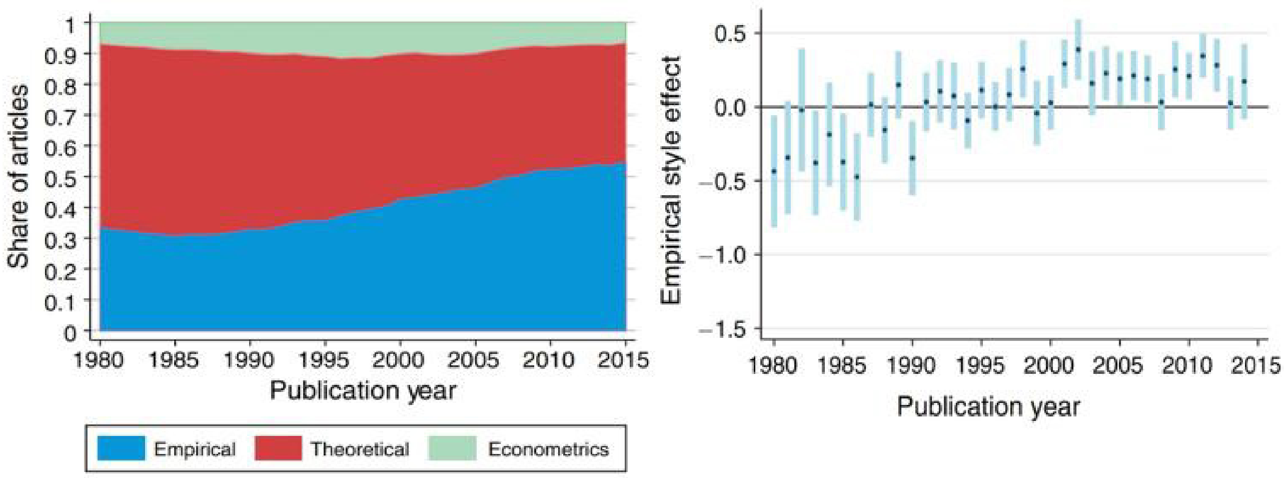

Figure 2: The publication of empirical and mathematical economics papers in the top economics journals and their influence on economic theory (based on Angrist et al., 2017)

3. Why Economics Needs Mathematics and The Problem of De- mathematizing in the Field of Economics

3.1 The core role of a mathematical model in economics

Mathematical models are powerful because they provide a logically consistent framework for economic theory. Arrow & Debreu (1954), in their pioneering work, pointed out that mathematical models help to form hypotheses and draw unambiguous conclusions, which are essential for advancing economic theory and research. A significant advantage of using mathematical language is its clarity and logical rigour. Mathematical expressions can succinctly and accurately convey ideas, reduce ambiguous expressions, and clarify theoretical hypotheses and derivation processes. In addition, mathematical logic can test the theory's consistency and completeness, ensuring there are no logical gaps between the previously mentioned conclusions.

3.2 Specific application of mathematical tools in economics

A famous example is the Black-Scholes Model, which revolutionized the field of financial economics by providing a robust framework for pricing options and other derivative securities. The mathematical basis of this model enables traders and financial institutions to make correct assessments of the risks and develop the right strategies for investments. The implementation of this model has been very effective in the world's financial system and has shown that mathematics is essential in today's economy.

3.3 Possible problems caused by de-mathematizing

Stripping economics of mathematical models and quantitative analysis means that the results are less accurate and less likely to be expected. Economic ideas are tough to test when they are not backed up with the right mathematical approach. According to Hansen (1982), simplification of models due to a lack of mathematical complexity can hamper research in terms of its accuracy, validity, reliability, and repeatability. Compared with other approaches that do not involve mathematics, for example, qualitative analysis, or techniques relying on empirical intuition, it can be possible to encounter certain constraints in addressing specific economic issues. Non-mathematical approaches are based on the subjective impressions of the researcher and their intuition and, therefore, the outcomes obtained cannot be easily reproduced. However, when data is extensive or when several factors are involved in a system, more than non-mathematical approaches may be required to give a more comprehensive understanding. This implies that the decision-making process may be more based on assumptions and guesswork than on fact-finding because there is no clear information to guide the decision-making process. This approach results in poor policy execution and has adverse repercussions including market distortions or resource wasting.

3.4 The value of mathematical tools to economic theory and practice

Mathematics is not only a means for the development of theory, but also the foundation for the application of theory. It is therefore important more than ever before to use mathematical tools for quantitative analysis in today's complex and unpredictable global economic environment. It is through mathematical models that economists can have a clearer view of the macroeconomic, financial market, and policy impact.

In macroeconomics, the IS-LM model IS a perfect example of how shifts in aggregate demand bring about changes in the levels of national income and rates of interest through the equilibrium of both the money and commodity markets. In microeconomics, the supply and demand curve shows the relationship between the price and quantity of a particular commodity in a market. In finance, we have the Capital asset pricing Model (CAPM) that seeks to establish how risk influences the expected returns.

These models are very important and they do help in enhancing the prediction accuracy. For instance, the CAPM model can make a more rational analysis of the investment worth of various stocks. Besides, it also helps in policy recommendations where the government can estimate the future economic growth rate for successive years through macroeconomic models to modify the fiscal policy. Furthermore, it has been observed that the findings of the studies on unemployment fluctuations have been enhanced greatly since the application of modern econometric methods.

Statistics show that the quality of papers has improved significantly over the past few decades as more sophisticated mathematical models have been developed and applied to economic research. For example, many articles published in the American Economic Review employ quantitative methods, and these articles are often cited more frequently than other types of research.

A typical success story is the development of the Black-Scholes option pricing formula and its widespread application in financial markets. This formula provides a solid theoretical basis for derivatives trading and dramatically promotes the growth of the financial derivatives market in the world. Another example is the prospect theory proposed by Daniel Kahneman, a Nobel laureate in economics. He used data from psychological experiments and rigorous mathematical modelling to reveal the behavioural biases of people in the face of risky decisions, which profoundly impacted the development of behavioural economics.

The work in both cases would have been challenging without the right mathematical tools. For the Black-Scholes model, the lack of effective calculation methods means that it is difficult to estimate the price of options accurately, which hinders the innovation and development of related financial products. Kahneman's work, however, would not have been as widely accepted as it is today if it had been written rather than mathematically expressed, nor would it have been easy to compare and verify with other academic work.

Therefore, maintaining the central position of mathematics in economics is indispensable for promoting the progress of the discipline, improving the quality of policymaking, and promoting global economic stability and development.

4. Future trends of mathematics in economics

In economics, the application of mathematics is not only a tool for academic research but also a pivotal force to promote the development of economic theory and practical policy making. With the rapid growth of big data, artificial intelligence, machine learning and other technologies, economic research methodology is undergoing unprecedented changes. Based on the current discussion, this paper will extend the discussion on the possibilities of mathematical applications in economics as well as the barriers that exist in order to come up with new ideas and future guidelines for economic researchers.

4.1 In-depth application of advanced mathematical methods

With the continuous progress of mathematics, higher-order mathematical tools such as stochastic processes, partial differential equations and nonlinear dynamic systems will be gradually introduced into the study of economics. These advanced mathematical methods can better describe the dynamic evolution process of complex economic systems and improve economic models' forecasting accuracy and explanatory power. For example, in macroeconomic policy analysis, the application of dynamic stochastic general equilibrium (DSGE) models has gradually become mainstream, which can more comprehensively consider the economic system's uncertainty and dynamic adjustment processes.

4.2 Integration of machine learning and artificial intelligence

In recent years, machine learning algorithms have become increasingly widely used in economics, especially when dealing with large data sets and complex nonlinear relationships. In the future, economics will be combined with artificial intelligence technology, and the economic laws behind the data will be mined through deep learning, reinforcement learning, and other methods to provide more accurate references for policymaking. For example, machine learning-based models have achieved remarkable results in predicting economic recessions, analyzing financial market fluctuations, and so on.

4.3 Deepening of Interdisciplinary Cooperation

Economics is a social science, and its research often involves multiple fields of knowledge. In the future, economics will pay more attention to cross-integration with other disciplines, such as physics, biology, computer science, etc. This interdisciplinary collaboration can provide new perspectives and methods for economic research and promote knowledge sharing and innovation among disciplines. For example, concepts and techniques such as complex system theory in physics and evolutionary game theory in biology are gradually being introduced into economic research, promoting economic theory development.

5. Conclusion

In economic research, adopting more straightforward mathematical methods and a growing tendency towards empirical research have reduced the reliance on mathematical analysis. However, advanced mathematical tools are often needed to understand and accurately model complex economic phenomena comprehensively. Given this, more than simple mathematical methods can undermine the depth and rigour of financial research. This so-called "de-mathematization" trend can lead to incomplete or misleading conclusions, thereby undermining the credibility of the field of economics and reducing the effectiveness of its practical applications.

The paper also emphasizes a complementary rather than competitive relationship between mathematics and economic theory. Translating complex theories into practical insights is necessary to close the gap between academic discourse and policy practice. To this end, financial professionals should devote themselves to using mathematical tools to promote the development of economic thought. By improving their mathematical ability, economists can understand economic theories more deeply and enhance their explanatory and predictive power, thus making economics a more solid and reliable scientific system. At the same time, combining modern computing techniques such as empirical analysis and machine learning can better use the advantages of mathematical tools to deepen our understanding of economic phenomena.

Table 1: Common Mathematics Applied across the Fields of Economics

Field of Economics	Economic theory and design	Mathematics in these papers	Represetative studies
Urban and regional economics	A spatial equilibrium model; hedonic pricing; urban land use theory; spatial sorting	Calculus; differential equa- tions; linear algebra; dynamic optimization	Allen and Arkolakis (2022); Jerch et al. (2024); Waxman et al. (2020)
Development, public and health economics	Causal inference: difference-in-differences; randomized controlled trials; synthetic control method; regression discontinuity; bunching regression	Parametric and non-parametric regressions	Abadie et al. (2010); Banerjee et al. (2007); Chen et al.(2021)
International economics	Gravity model; structural general equilibrium model	Linear regression; linear and dynamic programing; numer- ical analysis; taylor series; mul- tivariable calculus	Allen (2014); Autor et al.(2020)

Labor economics	Causal inference; Roy model; Push-pull model	Linear regression; Lagrangian optimization	Anderson (2011); Borjas (2003); Card and Krueger (1993); Taber and Vejlin(2020)
Environmental and resouces Economics	Causal inference; data envelopment analysis	Linear regression; nonparamet- ric method	Barreca et al. (2016); Currie et al. (2015); Deschenes et al.(2017)
Bevioural economics	Game theory	Topology ; probabilty theory	Anbarci and Sun(2013); Thaler(2018)
Financial and industrial Economics	General equilibrium model; linear and dynamic programing	Stochastic processes; real ana- lysis	Holmstrom and Tirole (1997); Lyasoff(2017)
Econometrics	Identifying average treatment effects; Regression analysis	Probabilty theory; machine learning; measure theory	Athey and Imbens (2017); Hirano et al. (2003)
Political and law economics	Causal inference; game theory	Topology, complex analysis, Linear regression	Arrow(1977); Cao et al. (2022); Donohue III and Levitt (2001); Levitt (2004)

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