The Kelly Criterion: Optimizing Decision-Making in Risk Management

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Abstract

The Kelly capital growth investment criterion, or Kelly criterion, defines the fraction of wealth to invest in a favorable investment opportunity such that the exponential growth rate is maximized. Maximizing the exponential growth rate is equivalent to maximizing logarithmic utility.[1] It all began in the mid-20th century when John L. Kelly Jr., a researcher at Bell Labs, developed the criterion as a solution to a problem related to the efficient transmission of information in telecommunications. Kelly introduced the concept of the Kelly Criterion within his paper "A New Interpretation of Information Rate," which he published in his early years. However, he did not refer to the Kelly Criterion by its now-known name. The Kelly Criterion initially gained recognition in academic and mathematical circles, primarily for its applications in information theory. Then, professional gamblers and investors started using the Kelly Criterion to manage their bankrolls and make more informed betting decisions. The financial industry also embraced the Kelly Criterion as an alternative approach to portfolio management and investment strategies. The Kelly Criterion gained further attention in investment circles, particularly in hedge funds and wealth management.

Keywords: Kelly Criterion, Mathematical Foundation, practical applications, limitations

2. Introduction

First, we discuss the Kelly Criterion's mathematical foundation, which is explained by the formula

$$f* = \frac{bp - q}{b}$$

Where f* is the amount that the person wants to use
Where b is the actual odds of the bet
Where p is the probability of winning the bet
Where q is the probability of losing the bet
The key idea behind the Kelly Criterion is to allocate more capital to opportunities with higher expected returns (higher p) and favorable odds (higher b) while also considering the possibility of losses. If the result of the formula is positive, it indicates that the opportunity is favorable and capital should be allocated to it. Conversely, the opportunity should be avoided if the result is negative or zero. The Kelly Criterion is like having a reliable compass to help you navigate the complex terrain of decision-making. It doesn't require diving deep into mathematical formulas or complex calculations. Instead, it provides a practical and intuitive method that anyone can use, regardless of their mathematical background.

In this research paper, we will introduce you to the world of the Kelly Criterion, breaking down its fundamental concepts and demonstrating its real-world applications. Whether you're a novice investor or a casual sports enthusiast, we aim to show you how the Kelly Criterion can be a valuable tool in your decision-making toolkit, helping you optimize your choices and achieve better outcomes without needing advanced math skills. By the end of this paper, you'll clearly understand how the Kelly Criterion can empower you to make more informed and rational decisions in the face of uncertainty.

3. Practical Applications

3.1 Gambling

The Kelly Criterion offers a mechanism for gamblers to choose the best wager size in order to optimize their anticipated long-term wealth increase. This mechanism involves many steps; the first step is estimating the expected value of the gambling opportunity. The expected value represents the average amount of money one can expect to win or lose per bet, considering the potential outcomes and their probabilities. The calculated optimal fraction (f*) indicates how much of your capital should be wagered on the gambling opportunity. If f* is positive, it suggests that the opportunity is favorable, and you should allocate the calculated fraction of your capital. If f* is negative or zero, the opportunity is unfavorable, and you should avoid betting on it. As your capital changes after each bet, the Kelly Criterion should be reapplied using the updated capital and the new estimated probabilities for subsequent bets. This iterative approach optimizes capital allocation for each bet, considering the changing circumstances. Individuals can lower their risk of significant losses while taking advantage of different situations by not overbetting. For instance, take a bettor

assessing the probability of a football team winning at 60%, and the odds offered are 2.0 (even money). Using the Kelly Criterion:

p=0.60
q=0.40
b=2.0
f* = = 0.2

$$f^* = \frac{(2.0*0.6)-0.4}{2.0} = 0.2$$

Given the perceived odds, the formula recommends betting 20% of the bankroll on the football team.

3.2 Investing

The Kelly Criterion is also useful in investing. Unlike traditional portfolio optimization techniques like mean-variance analysis, which focuses solely on expected returns and standard deviations, the Kelly Criterion considers the entire distribution of returns. Investors must begin by assessing the probability of success for each investment opportunity. This assessment requires a comprehensive analysis of factors such as market conditions, industry trends, and the financial health of the companies involved. Accurate probability estimation is crucial for the Kelly Criterion to yield optimal results. Investors must also evaluate each investment opportunity's potential returns and risks. This includes considering expected returns, volatility, and correlation with existing portfolio holdings. It considers the shape and skewness of the distribution, leading to more realistic and robust investment decisions. The Kelly Criterion is versatile and can be applied to various asset classes, including stocks, bonds, real estate, and alternative investments. It provides a unified framework for managing diverse investment portfolios.

4. Limitations and Extensions

4.1 Limitations

The effectiveness of the Kelly Criterion hinges on the investor's ability to accurately estimate probabilities. Inaccurate assessments can lead to suboptimal outcomes. Markets are dynamic, and investment conditions can change rapidly. Investors must continuously reassess their investments and adjust their capital allocations accordingly. The formula also ultimately assumes that the probabilities and odds remain constant, which may not be true.

4.2 Fractional Kelly (Extension of the Kelly Criterion)

The risk-taking suggested by growth optimal strategies is often perceived as high, meaning that the wealth path is somewhat rough and the volatility of wealth levels over time is high. The Fractional Kelly Criterion is a modified version of the original Kelly Criterion. While the original Kelly Criterion calculates the optimal fraction of your capital to bet or invest, the Fractional Kelly Criterion recommends betting or investing only a fraction of the optimal amount determined by the Kelly formula. The primary motivation for using the Fractional Kelly Criterion is to reduce the risk of large losses and provide a more conservative approach to capital allocation. By betting or investing a fraction of the recommended amount, individuals can lower their portfolio's volatility and potential drawdowns, making it more suitable for risk-averse investors. The Fractional Kelly Criterion allows investors and gamblers to balance optimizing returns (as suggested by the original Kelly Criterion) and managing risk by betting or investing a smaller portion of their capital. This approach is particularly useful when individuals are risk-averse or want to protect their capital from significant losses while benefiting from the mathematical edge provided by the Kelly Criterion. It can be much better in some cases than the actual Kelly Criterion.

5. Conclusion

The Kelly Criterion has proven to be a fundamental concept in investment and risk management. Its elegant mathematical formulation provides a systematic approach to capital allocation that maximizes expected logarithmic wealth. Despite its limitations, the Kelly Criterion remains a valuable tool for individuals and organizations seeking to make rational decisions under uncertainty. Its diverse applications in finance, gambling, and information theory underscore its enduring significance in modern decision-making processes. As technology advances, the Kelly Criterion's relevance and influence will likely persist, shaping how we approach risk and reward in an ever-changing world.

Citations:

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