# The Application of Markowitz Model and Index Model in the Real Financial Market

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**Abstract:**

This essay aims to analyze the optimal capital allocation in a ten-stock portfolio. The research paper will delve into two big aspects: the introduction of the Markowitz Model ("MM") and Index Model ("IM") and the implementation of MM and IM. A recent 20 years of historical daily total return data for ten different stocks, which belong to groups of three to four different sectors (according to Yahoo Finance), one (S&P 500) equity index (a total of eleven risky assets) and a proxy for risk-free rate (one-month Fed Funds rate) will be used. Utilizing these optimization inputs for the Markowitz Model and Index Model, I will find the regions of permissible portfolios (efficient frontier, minimal risk portfolio, optimal portfolio, and minimal return portfolios frontier) for the five cases of the additional constraints.

**Keywords:** Markowitz Model, index Model, real Financial Market, capital

## Introduction

This research paper will be divided into 5 sections. The first section will give a brief introduction to the research paper. The second section will introduce the Markowitz Model, the Index Model, and the five constraints. The third section will introduce the ten companies' selection and the corresponding data analysis. The fourth section will apply the Markowitz Model and Index Model to determine the optimal weights of different stocks under five constraints. The fifth section will generalize the results of MM and IM under the five constraints. It will analyze the differences and common places between the two models.

## Theoretical Models

**Markowitz Model.**

Markowitz Model, also known as the mean-variance model, was first introduced by Harry Markowitz-the master of finance-in 1952. Generally, the Markowitz Model uses the expected return ("E(r)") and the standard deviation ("σ") of each asset and the correlations between each asset to calculate the investor's portfolio's efficient frontier. Then, it combines the efficient frontier and the investor's capital allocation line (CAL) to find the optimal portfolio:

Assume there are n risky assets in the financial market; the return rate of each asset is $r_1$ , $r_2$ , ... $r_n$ . The weight of each asset is $\omega_1$ , …, $\omega_n$ . Then the total return of the portfolio equals to

$$r_p = \sum_{i=1}^{n} \omega_i r_i \text{ .Where } \sum_{i=1}^{n} \omega_i = 1$$

Thus, the formula for the total expected return is

$$E(r_p) = \sum_{i=1}^{n} \omega_i E(r_i)$$

The variance is

$$\mathrm{Var}(r_p) = \sum_{i=1}^{n} \omega_i^2 \mathrm{Var}(r_i) + \sum_{i \neq j} \omega_i \omega_j \mathrm{Cov}(r_i, r_j)$$

The premise of the investor's capital allocation is that the investor has already formed the expectation, which is the probability distribution of $r_1$ to $r_n$ , then sets the goal and eventually confirms the weights of different assets.

Assume the original purchasing power of an investor is . The future purchasing power will be $W_0$ *(1+ $r_p$ ) if the investor's utility is a function of utility and wealth, which means the utility only depends on the investor's wealth, then the utility U(r) is a random variable. From the perspective of maximizing utility, the procedure of the investor making an investment decision can be displayed through the following formula:

$$\max_{\omega_i} E\left[U\left(W_0 r_p\right)\right]$$

$$\text{s.t} \sum_{i=1}^{n} \omega_i = 1$$

The equation can be transformed into:

$$\max_{\omega_i} E\left[U\left(r_p\right)\right]$$

$$\text{s.t} \sum_{i=1}^{n} \omega_i = 1$$

U( $r_p$ , σ)=E(r)-(A*2, A denotes the index of risk aversion of the investor.

Using Taylor expansion on E[U()], we can easily tell that to exactly follow the normal distribution; thus, the expected utility depends on the portfolio's mean and variance. We again assume that U(r) is a concave function, then the procedure of investing is:

$$\min_{\omega_i} \mathrm{Var}\left(r_p\right)$$

$$\text{s.t} \begin{cases} \mathrm{r_p} = \sum_{i=1}^{n} \omega_i \mathrm{E}\left(\mathrm{r_p}\right) \\ \sum_{i=1}^{n} \omega_i = 1 \end{cases}$$

Equation 1 can be solved by Lagrange method:

$$\begin{pmatrix} \sum & e & r \\ e' & 0 & 0 \\ r' & 0 & 0 \end{pmatrix} \begin{pmatrix} \omega \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ r_p \end{pmatrix}$$

In the matrices above, the column matrix made up of the expected returns of the n assets, Σ is the covariance matrix of return rates, and e is the unit matrix of n * 1. The ultimate version of the optimal weight is:

$$\omega^* = a + br_p$$

As we can see, there is a linear relationship between the optimal choice and the expected return. The risk Var( $r_p$ ) is the quadratic function form of ω.

## 2. Single-Index Model

The single index model, which simplifies the estimation of the covariance matrix problem, was proposed by William F Sharpe. To illustrate, assume we are analyzing 50 stocks. The list of the Markowitz Model should include the following:

1) 50 estimates of expected returns;
2) 50 estimates of variances;
3) 1,225 estimates of covariances;

Total=1325 estimates.

Repeating this for 100 stocks will lead to 5150 estimates. Also, the Markowitz Model tells nothing about how to produce those estimates other than calculating the historical data average. Still, past returns are unreliable since the unpredictable financial environment keeps changing.

First, decompose security i returns into:

$$r_i = \mathrm{E}\left(r_i\right) + e_i$$

Where the unexpected return has zero mean: E $\left(\overline{e_l}\right)$ = 0 and a standard deviation of , the uncertainty is firm-specific: E() = 0 for all i ≠ j.

Further, assume that are normally distributed.

Next, assume that there exists a common, stock-independent, "macroeconomic" random factor m, which equally influences all stocks:

$$r_i = E\left(r_i\right) + m + e_i,$$

M is also normally distributed, its standard deviation is , and E( $r_i$ *m)=0.

$$\sigma_i^2 = \sigma_m{}^2 + \sigma^2\left(e_i\right)$$

Then:

$$\mathrm{Cov}\left(r_i, r_j\right) = \sigma_m{}^2 .$$

Finally, we need to account that some firms are more and some less dependent on the macroeconomic factor:

$$r_i = E\left(r_i\right) + \beta_i * m + e_i$$

The risk and covariance are determined by the stock's β-coefficient:

$$\sigma_i{}^2 = \beta_i^2 * \sigma_m{}^2 + \sigma^2\left(e_i\right), \mathrm{Cov}\left(r_i, r_j\right) = \beta_i * \beta_j * \sigma_m{}^2$$

Correlation = Product of correlations with the market index:

$$\mathrm{Corr}\left(r_i, r_j\right) = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j}$$

-the securities betas and the properties of the market index determine all of these.

The most convenient action is to choose a broad index (S&P 500) as a broad macroeconomic factor. It has a considerable amount of past data to be used for estimation. If M denotes the market index, its excess return is $R_M = r_M - r_f$ and the standard deviation $\sigma_M$ . The factor β can be estimated using linear regression between observations of $R_i(t)$ and $R_M(t)$ :

$$R_i(t) = \alpha_i + \beta_i * R_M(t) + e_i(t).$$

If we take the expected value of both sides, we get:

$$\mathrm{E}(R_i) = \alpha_i + \beta_i * E\left(R_M\right),$$

Where the first term, $\alpha_i$, is non-market risk-premium.

Then, we consider the Index Model on the portfolio level. Assume, for simplicity, an equally-weighted portfolio $w_i = \frac{1}{n}$ :

$$R_p = \frac{1}{n}\sum_{i=1}^{n} \alpha_i + \left(\frac{1}{n}\sum_{i=1}^{n} \beta_i\right) * R_m + \frac{1}{n}\sum_{i=1}^{n} e_i,$$

From which follows:

$$\beta_p = \frac{1}{n}\sum_{i=1}^{n} \beta_i, \alpha_p = \frac{1}{n}\sum_{i=1}^{n} \alpha_i, \text{and } e_p = \frac{1}{n}\sum_{i=1}^{n} e_i.$$

The portfolio's variance is:

$$\sigma_p{}^2 = \beta_p{}^2 \sigma_m{}^2 + \sigma^2\left(e_p\right).$$

The firm-specific risk can be diversified:

$$\sigma^2\left(e_p\right)=\sum_{i=1}^{n}\left(\frac{1}{n}\right)^2\sigma^2\left(e_i\right)=\frac{1}{n}\bar{\sigma}^2\left(e\right).$$

## 3. The Five Constraints

To simulate the financial market as realistically as possible, I set five constraints to the model to simulate five different market environments. Then, the portfolios corresponding to the minimum variance and the maximum Sharpe ratio will be found and compared between the MM and IM models.

Constraint 1:

$$min\,\sigma_p,\ \text{s.t}\sum_{i=1}^{11}\left|w_i\right|\le 2$$

$$\text{max sharpe} = \frac{E\left(r_p\right)-r_f}{\sigma_p},\ \text{s.t}\sum_{i=1}^{11}\left|w_i\right|\le 2$$

Constraint 2:

$$\text{min }\sigma_p,\ \text{s.t}\left|w_i\right|\le 1,\ \text{for any i}$$

$$\text{max sharpe} = \frac{E\left(r_p\right)-r_f}{\sigma_p},\ \text{s.t}\left|w_i\right|\le 1,\ \text{for any i}$$

Constraint 3:

$$\text{min }\sigma_p,\ \text{no constraints}$$

$$\text{max sharpe} = \frac{E\left(r_p\right)-r_f}{\sigma_p},\ \text{no constraints}$$

Constraints 4:

$$\text{min }\sigma_p,\ \text{s.t } w_i\ge 0,\ \text{for any i}$$

$$\text{max sharpe} = \frac{E\left(r_p\right)-r_f}{\sigma_p},\ \text{s.t } w_i\ge 0,\ \text{for any i}$$

Constraint 5:

$$\text{min }\sigma_p,\ \text{s.t } w_1=0$$

$$\text{max sharpe} = \frac{E\left(r_p\right)-r_f}{\sigma_p},\ \text{s.t } w_1=0$$

## Companies and Data

### Introduction of the selected companies

#### Consumer Cyclical

1. Amazon.com, Inc.

Amazon Inc., with 2022 revenue of $514 billion, is one of the largest online e-commerce companies in the United States, based in Seattle, Washington. Amazon is one of the earliest companies to operate e-commerce on the Internet. Founded in 1994, Amazon initially only operated the online book sales business, and now it has expanded to a wide range of other products. It has become the online retailer with the largest variety of commodities in the world and the second-largest Internet enterprise. It also includes subsidiaries like Alexalnternet, a9, lab126, and Internet Movie Database (IMDB). Technology Company

2. Apple Inc.

Apple Inc. is an American high-tech company. In fiscal year 2021, Apple's revenue reached $365.8 billion. It was founded by Steve Jobs, Steve Gary Wozniak, and Ronald Gerald Wayne on April 1, 1976, and named Apple Computer Inc. On January 9, 2007, the company changed its name to Apple Inc., headquartered in Cupertino, California.

3. Citrix System, Inc.

Citrix is a high-tech enterprise dedicated to cloud computing virtualization, virtual desktops, and remote access technology. The now popular BYOD (Bring Your Device) is the idea of Citrix Company. In 1997, Citrix established the development vision of "making information access as simple and convenient as making a phone call, so that anyone can get it at any time and anywhere at any time." This concept is the prototype of today's mobile office. With the rapid development of Internet technology, through the virtual desktop based on cloud computing technology, People can use any device to access their work environment at any time, anywhere, and seamlessly switch between various scenarios, making the office ubiquitous and easy to do.

## Financial Service

1. JPMorgan Chase & Co.

J.P. Morgan is a world-renowned comprehensive financial company that mainly provides commercial banking, investment banking, and other financial services. The company's asset size ranks in the top 20 of the famous financial magazine "Fortune 500 large companies in the United States and is one of the highest credit rating companies among global financial institutions. Morgan's commercial banking subsidiary, Morgan Guarantee Trust Company of New York, is the only U.S. commercial bank with a triple-A credit rating.

2. Berkshire Hathaway Inc.

Berkshire Hathaway, founded in 1956 by Warren Buffett, is an insurance company with business activities in many other areas. The most important business is property and casualty insurance based on direct premiums and reinsurance amounts. Berkshire Hathaway has many subsidiaries, including the GEICO Company, the sixth-largest auto insurer in the United States; General Re is one

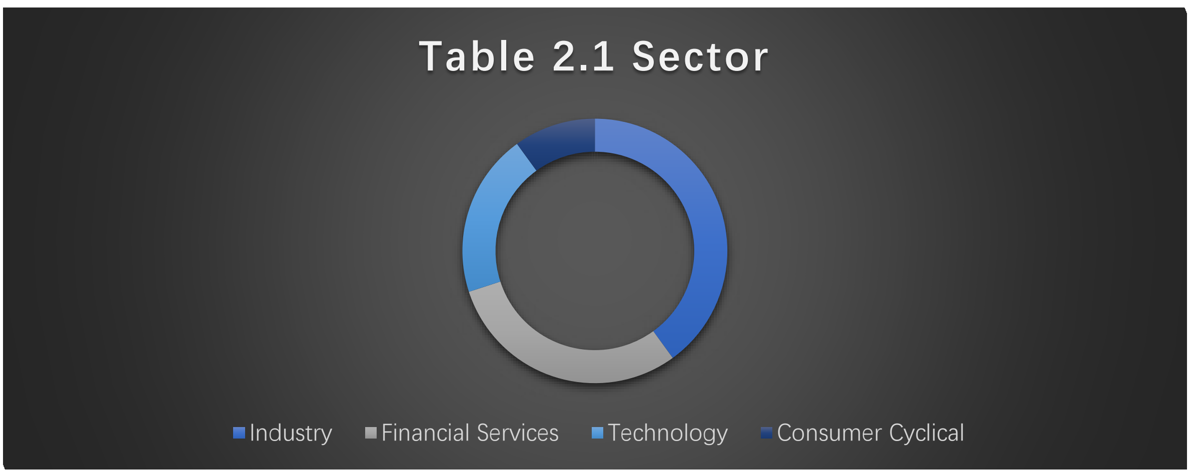
of the four largest reinsurance companies in the world.
3. The Progressive Corporation
Progressive Insurance (NYSE: PGR) - as its name says - has been an innovative force in the U.S. auto insurance industry since it was founded in 1937 by Jack Green and Joe Lewis.

## Industry

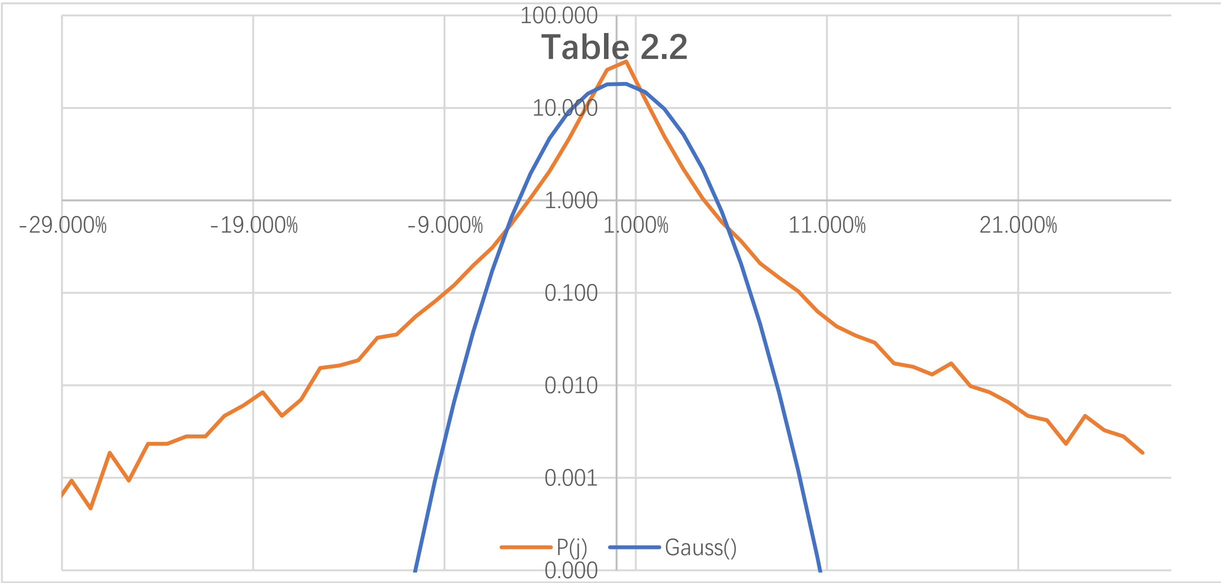
1. United Parcel Service, Inc.
UPS (United Parcel Service, Inc.) Founded in 1907 and headquartered in Atlanta, Georgia, United Parcel Service (UPS) is a global leader in logistics, providing package and cargo transportation, international trade facilitation, advanced technology deployment, and various solutions designed to improve the efficiency of global business management.
2. FedEx Corporation
FedEx is an international express delivery group providing overnight express, ground express, heavy cargo delivery, document copying, and logistics services. It is headquartered in Memphis, Tennessee, United States, and is part of FedEx Corp. On February 6, 2009, FedEx opened its new Asia Pacific hub at Guangzhou Baiyun International Airport in China, which will be its hub for the entire Asia Pacific region for 30 years. On December 16, 2014, FedEx agreed to acquire reverse logistics company Genco. That represents a big push into e-commerce. In July 2020, the Forbes 2020 Global Brand Value 100 was released, and FedEx ranked 99th.
3. J.B. Hunt Transport Service, Inc.
J.B. Hunt Transport Services, Inc. (NASDAQ: JBHT) is a trucking and transportation company that Johnnie Bryan Hunt founded. It started with only five trucks and seven refrigerated trailers to support the original rice hull business. By 1983, J.B. Hunt had grown into the 80th largest trucking firm in the U.S. and earned $63 million in revenue. At that time, J.B. Hunt operated 550 tractors 1,049 trailers, and had roughly 1,050 employees.
This company has grown into one of the largest truckload transportation companies in the United States, with annual revenues of over $ 2 billion. The company primarily operates large semi-trailer trucks and provides transportation services throughout the continental United States, Canada, and Mexico. The company currently employs over 16,000 employees and operates more than 11,000 trucks. Over 47,000 trailers and containers can be found in the company’s fleet.
4. Landstar System, Inc.
Landstar Systems Inc. is an American company. In May 2022, Landstar Systems ranked 491st on the 2022 Fortune 500 list with revenues of 6,540.4 US dollars. In June 2023, with 7,439.7 (million U.S. dollars) in revenue, it was listed in the 2023 Fortune 500 list, ranking 489.

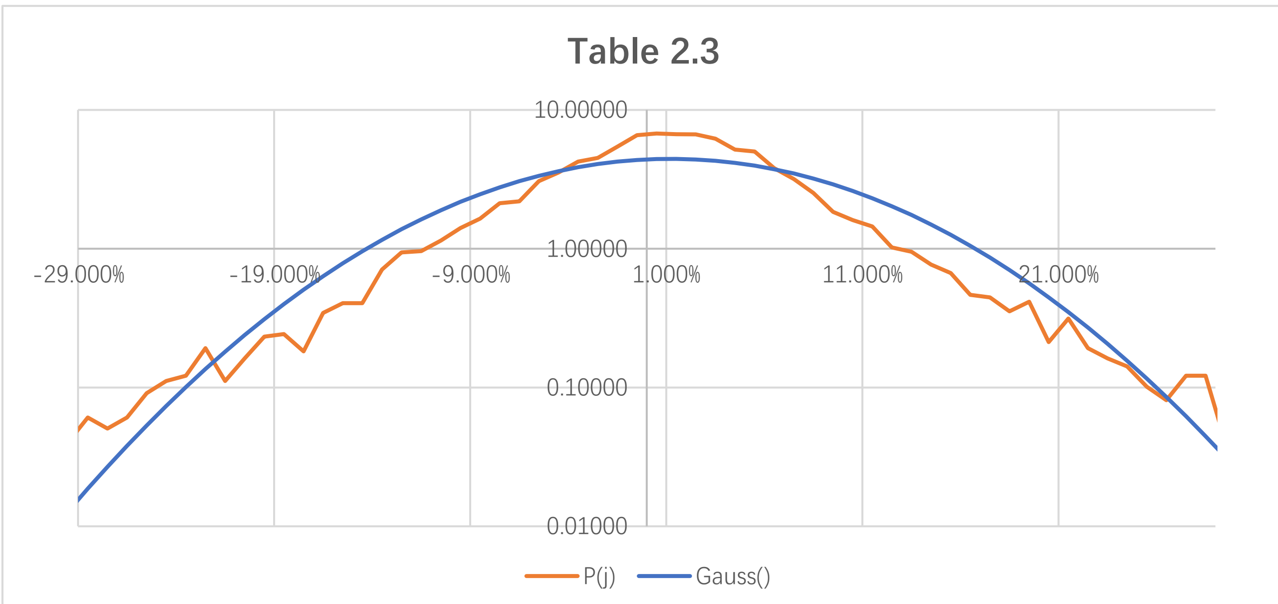
## 2. Data Analysis

As Table 2.1 shows, among the whole portfolio, 10% of the companies are consumer cyclical, 20% are in the technology field, 30% belong to the financial services sector, and the rest are industrial companies. Investing in companies that belong to different sectors can lower the correlation between them, effectively reducing the risk.




After getting the historical daily price data of the ten companies, I found some days don’t have any data because of holidays or other situations. Therefore, I transferred the raw daily data into five-day week daily data. I transferred the daily data into monthly data to reduce the non-Gaussian effect. TableTable 2.2 and Table 2.3 show that monthly data deviate from the normal distribution less than daily data.




**The Application of The Markowitz Model And The Index Model**

After processing the raw data, I calculate the annual average returns, standard deviations, betas, alphas, and residual deviations of the ten stocks based on the historical data displayed in Table 2.4.

**Table 2.4**

| | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Annual Average Return | 7.5% | 33.8% | 34.0% | 15.6% | 11.9% | 9.0% | 15.4% | 9.8% | 13.0% | 22.5% | 17.4% |
| Annual StDev | 14.9% | 41.4% | 34.5% | 41.5% | 29.0% | 16.2% | 21.1% | 21.4% | 26.7% | 30.7% | 23.9% |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| beta | 1.0000 | 1.3513 | 1.2569 | 1.2206 | 1.3608 | 0.5721 | 0.7120 | 0.8296 | 1.1036 | 1.0763 | 0.7975 |
| alpha | 0.0000 | 0.2360 | 0.2454 | 0.0643 | 0.0160 | 0.0469 | 0.1004 | 0.0359 | 0.0463 | 0.1441 | 0.1137 |
| residual Stdev | 0.0% | 36.2% | 29.0% | 37.3% | 20.8% | 13.8% | 18.2% | 17.5% | 21.1% | 26.2% | 20.8% |

Then, I calculate the correlations between the ten companies, finally depicting a covariance matrix as shown in Table 2.5

| | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SPX | 100.0% | 48.5% | 54.2% | 43.7% | 69.7% | 52.3% | 50.2% | 57.5% | 61.4% | 52.1% | 49.5% |
| AMZN | 48.5% | 100.0% | 37.7% | 21.7% | 25.2% | 11.8% | 20.0% | 29.6% | 28.0% | 30.8% | 25.6% |
| AAPL | 54.2% | 37.7% | 100.0% | 33.2% | 24.4% | 17.3% | 24.0% | 23.1% | 33.0% | 26.8% | 28.7% |
| CTXS | 43.7% | 21.7% | 33.2% | 100.0% | 32.4% | 18.1% | 27.1% | 26.4% | 33.1% | 29.0% | 25.2% |
| JPM | 69.7% | 25.2% | 24.4% | 32.4% | 100.0% | 45.2% | 39.3% | 36.1% | 44.0% | 44.2% | 37.5% |
| BRK/A | 52.3% | 11.8% | 17.3% | 18.1% | 45.2% | 100.0% | 26.4% | 40.4% | 38.5% | 23.9% | 23.4% |
| PGR | 50.2% | 20.0% | 24.0% | 27.1% | 39.3% | 26.4% | 100.0% | 39.2% | 36.5% | 28.0% | 28.9% |
| UPS | 57.5% | 29.6% | 23.1% | 26.4% | 36.1% | 40.4% | 39.2% | 100.0% | 67.5% | 45.9% | 44.1% |
| FDX | 61.4% | 28.0% | 33.0% | 33.1% | 44.0% | 38.5% | 36.5% | 67.5% | 100.0% | 53.7% | 48.2% |
| JBHT | 52.1% | 30.8% | 26.8% | 29.0% | 44.2% | 23.9% | 28.0% | 45.9% | 53.7% | 100.0% | 59.0% |
| LSTR | 49.5% | 25.6% | 28.7% | 25.2% | 37.5% | 23.4% | 28.9% | 44.1% | 48.2% | 59.0% | 100.0% |

Observing the correlations, it can be found that the correlation between companies that belong to different sectors is always relatively small. After doing all these, I implement the Markowitz Model and Index Model under the five constraints:

Constraint 1

Under constraint 1: $min\sigma_p$, s.t $\sum_{i=1}^{11}|w_i|\leq 2$; max Sharpe $=\frac{E(r_p)-r_f}{\sigma_p}$, s.t $\sum_{i=1}^{11}|w_i|\leq 2$, the return rate, standard deviation, and Sharpe ratio under the two situations—minimum variance and maximum return—of MM and IM are shown as follow:

**Table MM under C1**

| MM (Constr1): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 72.24% | -2.35% | -3.85% | -1.04% | -18.47% | 36.21% | 13.91% | 3.43% | -10.28% | -0.55% | 10.75% | 7.15% | 12.24% | 0.584 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -48.25% | 16.40% | 30.02% | -0.10% | -0.09% | 41.31% | 32.96% | -0.02% | -1.46% | 12.50% | 16.73% | 26.42% | 18.69% | 1.413 | 2.5 | 66.0% | 46.7% |

Graphically:

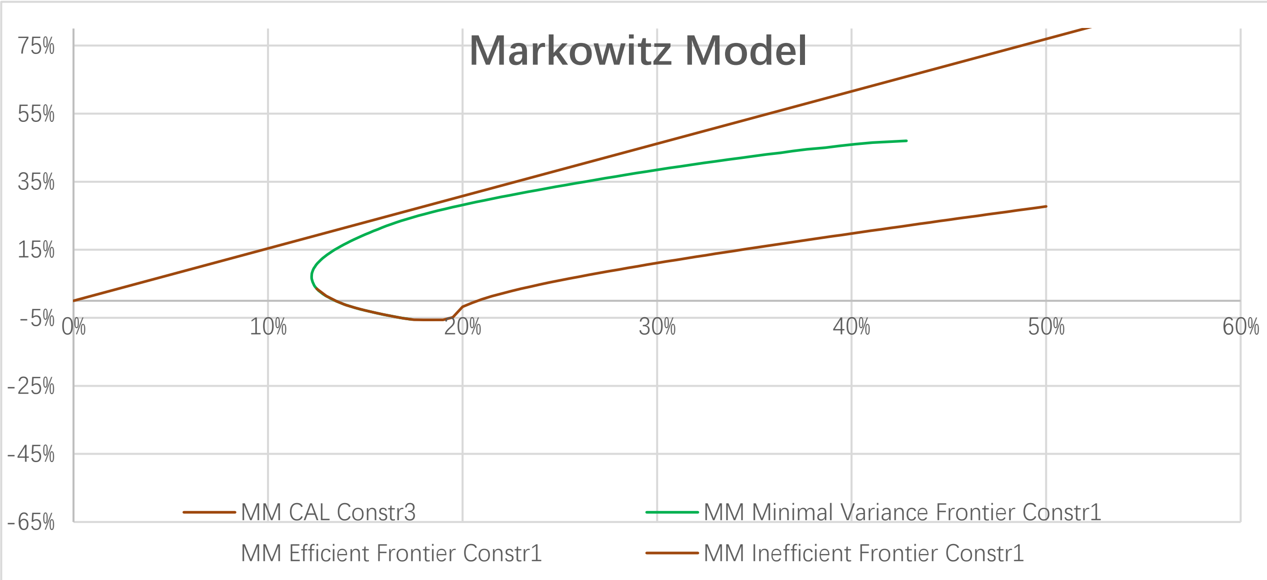
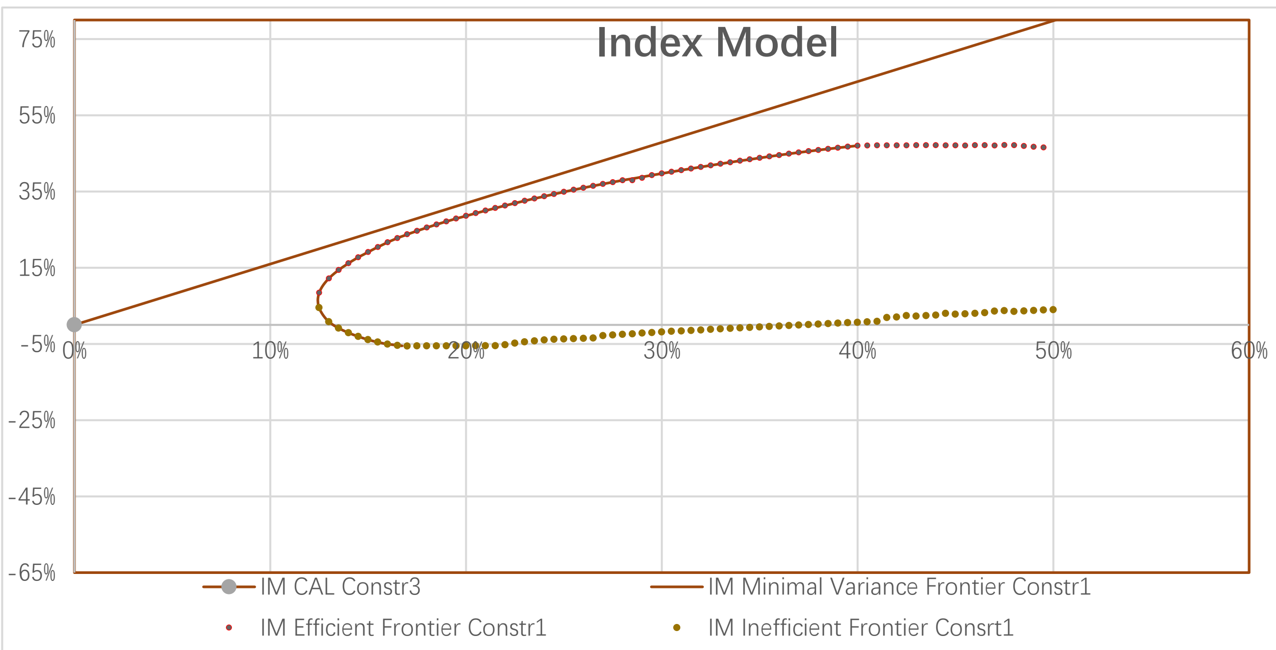



Table IM under C1

| IM (Constr1): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 65.79% | -4.14% | -4.74% | -2.44% | -12.87% | 34.51% | 13.42% | 8.56% | -3.61% | -1.72% | 7.24% | 6.46% | 12.43% | 0.520 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -47.30% | 17.84% | 30.52% | 0.50% | -2.68% | 22.44% | 31.53% | 1.17% | 0.11% | 19.20% | 26.66% | 28.57% | 19.98% | 1.430 | 2.5 | 71.4% | 50.0% |

Graphically:




Constraint 2

Under constraint 2: min $\sigma_p$, s.t $|w_i| \le 1$, for any I; max Sharpe $= \dfrac{E(r_p) - r_f}{\sigma_p}$, s.t $|w_i| \le 1$, for any I, the return rate, standard deviation, and Sharpe ratio under the two situations—minimum variance and maximum return—of MM and IM are shown as follow:

Table MM under C2

| MM (Constr2): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 72.24% | -2.35% | -3.85% | -1.04% | -18.47% | 36.21% | 13.91% | 3.43% | -10.28% | -0.55% | 10.75% | 7.15% | 12.24% | 0.584 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -100.00% | 22.33% | 39.76% | -1.20% | -0.50% | 62.48% | 46.01% | -3.11% | -10.56% | 20.88% | 23.91% | 33.18% | 22.11% | 1.501 | 2.5 | 83.0% | 55.3% |

Graphically:

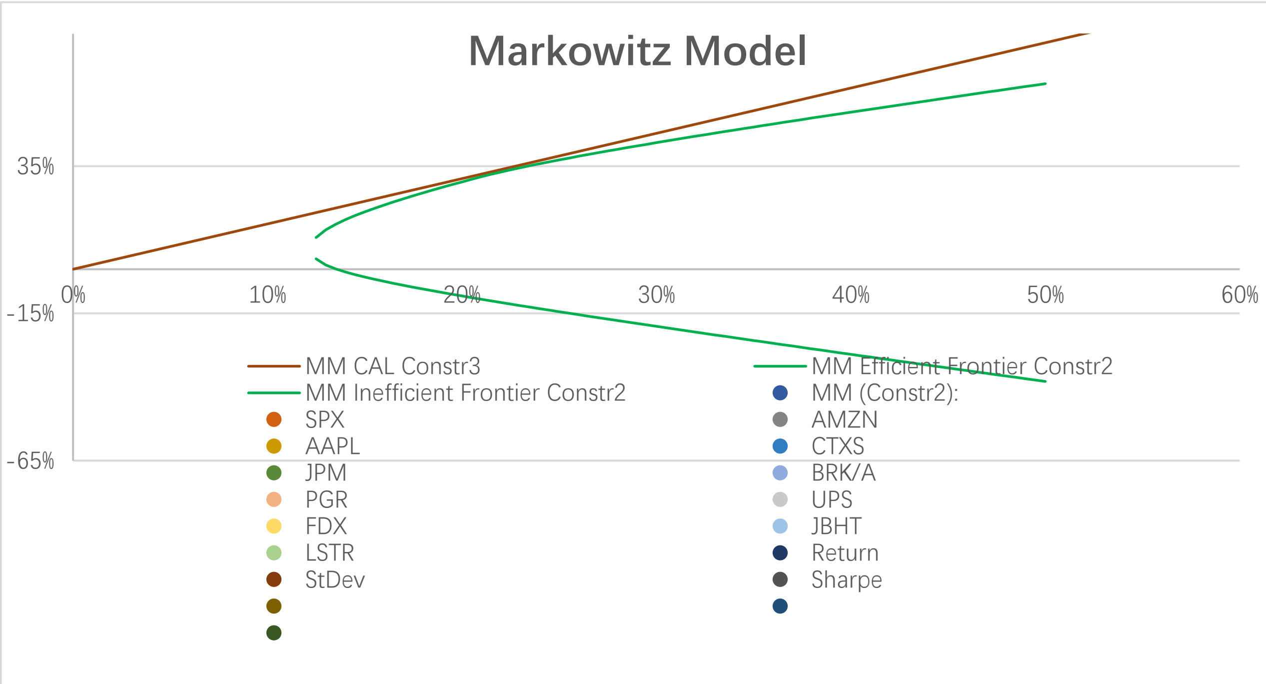
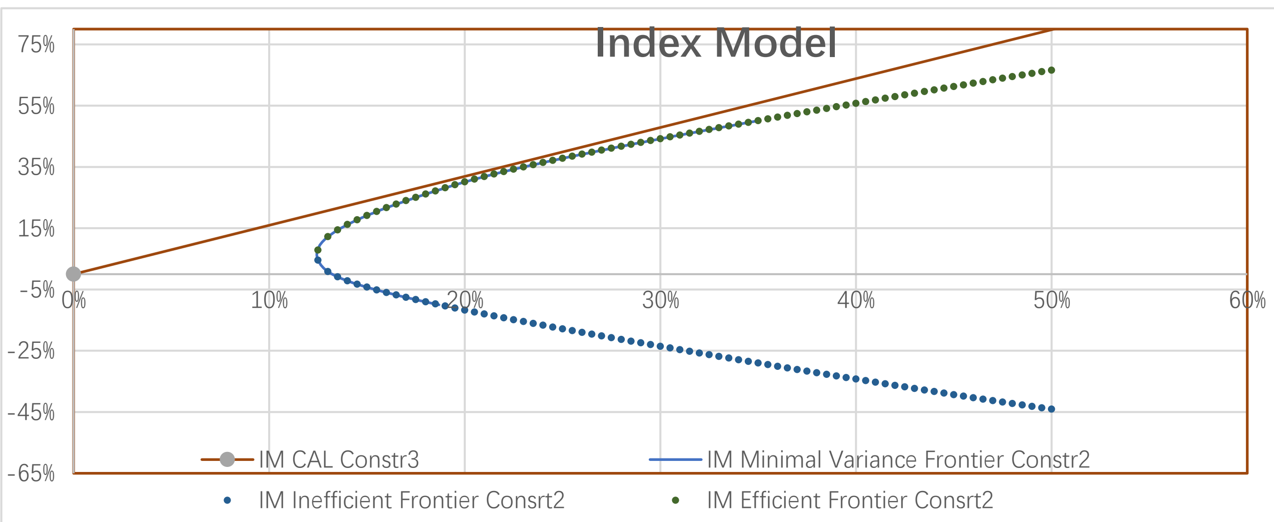



Table IM under C2

| IM (Constr2): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 65.79% | -4.14% | -4.74% | -2.44% | -12.87% | 34.51% | 13.42% | 8.56% | -3.61% | -1.72% | 7.24% | 6.46% | 12.43% | 0.520 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -100.00% | 21.85% | 36.62% | 3.09% | -8.79% | 33.32% | 39.98% | 9.52% | 5.62% | 24.97% | 33.83% | 34.07% | 22.40% | 1.521 | 2.5 | 85.2% | 56.0% |

Graphically:




**Constraints 3-5**

Similarly, I drew the chart of MM and IM under constraints 3-5 and compared them. Their tables and graphs are shown:

Table MM under C3-5

| MM (Constr3): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 72.24% | -2.35% | -3.85% | -1.04% | -18.47% | 36.21% | 13.91% | 3.43% | -10.28% | -0.55% | 10.75% | 7.15% | 12.24% | 0.584 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -237.52% | 37.06% | 65.39% | 0.42% | 17.30% | 91.59% | 68.19% | 1.27% | -8.69% | 30.97% | 34.02% | 49.61% | 32.25% | 1.539 | 2.5 | 124.0% | 80.6% |
| MM (Constr4): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 38.47% | 0.00% | 0.00% | 0.00% | 0.00% | 38.49% | 14.23% | 0.72% | 0.00% | 0.00% | 8.09% | 10.04% | 13.09% | 0.767 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 12.96% | 25.21% | 0.00% | 0.00% | 19.26% | 22.72% | 0.00% | 0.00% | 8.81% | 11.04% | 22.09% | 17.62% | 1.254 | 2.5 | 55.2% | 44.1% |
| MM (Constr5): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 0.00% | 2.45% | 4.19% | 0.84% | -7.09% | 56.31% | 23.52% | 11.41% | -8.05% | 0.60% | 15.82% | 13.20% | 13.39% | 0.986 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 14.65% | 26.73% | -3.44% | -15.54% | 36.35% | 31.99% | -12.17% | -13.21% | 17.83% | 16.81% | 23.89% | 18.02% | 1.326 | 2.5 | 59.7% | 45.0% |

Graphically:

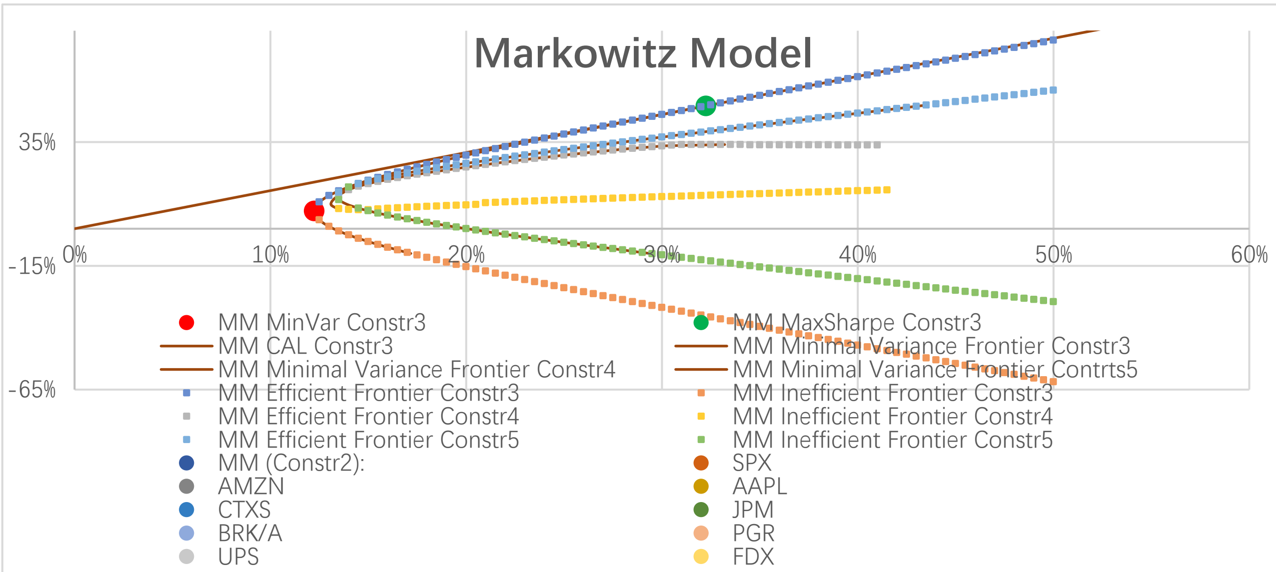
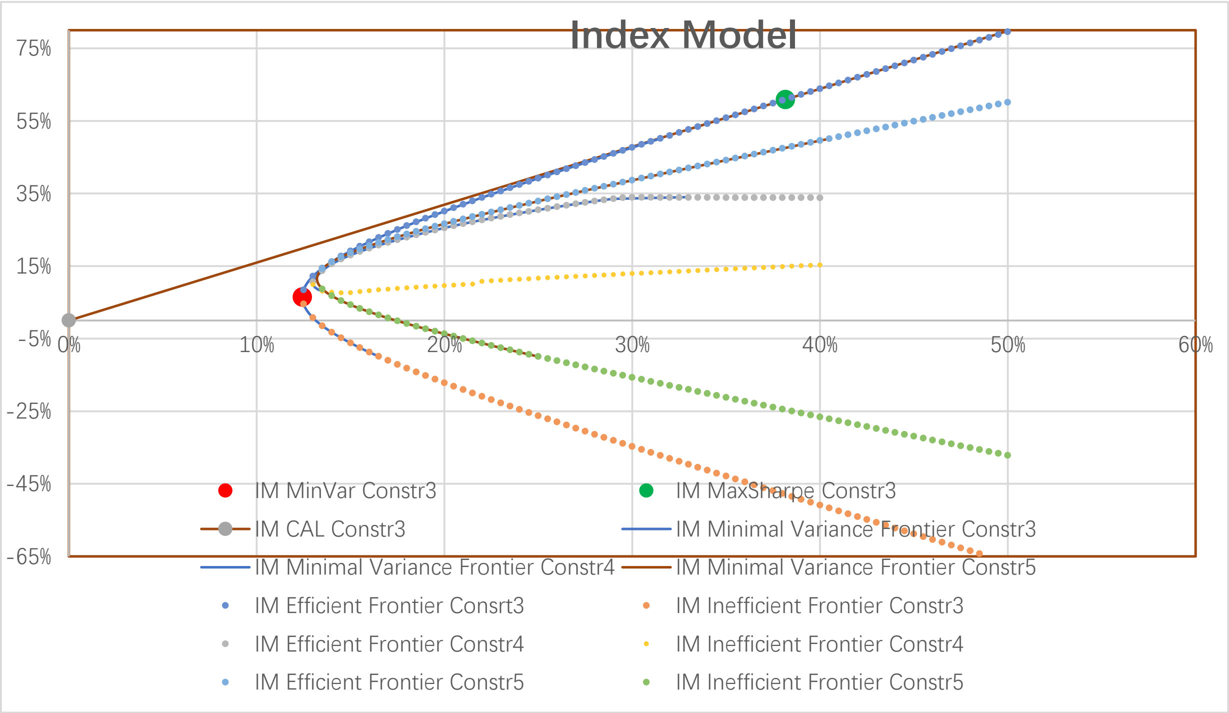



Table IM under C3-5

| IM (Constr3): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 65.79% | -4.14% | -4.74% | -2.44% | -12.87% | 34.51% | 13.42% | 8.56% | -3.61% | -1.72% | 7.24% | 6.46% | 12.43% | 0.520 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -329.78% | 43.02% | 69.96% | 11.02% | 8.83% | 58.47% | 72.35% | 27.91% | 24.94% | 50.34% | 62.93% | 60.91% | 38.16% | 1.596 | 2.5 | 152.3% | 95.4% |
| IM (Constr4): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 30.46% | 0.00% | 0.00% | 0.00% | 0.00% | 37.65% | 14.64% | 9.34% | 0.00% | 0.00% | 7.90% | 10.24% | 12.99% | 0.788 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 15.46% | 27.12% | 0.00% | 0.00% | 3.26% | 21.36% | 0.00% | 0.00% | 13.98% | 18.82% | 24.46% | 19.16% | 1.277 | 2.5 | 61.1% | 47.9% |
| IM (Constr5): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 0.00% | -1.55% | -0.81% | -0.11% | -5.02% | 47.99% | 21.65% | 17.82% | 3.45% | 2.82% | 13.76% | 11.47% | 13.21% | 0.869 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 18.56% | 31.69% | -0.47% | -21.23% | 11.67% | 27.66% | -5.30% | -5.40% | 18.57% | 24.25% | 26.95% | 20.24% | 1.331 | 2.5 | 67.4% | 50.6% |

Graphically:




**Evaluation and Comparison of the Markowitz Model and Index Model**

The differences in the above results of each constraint are because of the differences between the Markowitz and Index Model. Markowitz Model and Index Model are both important financial concepts, particularly portfolio management. Here are the differences and commonalities between the two:

Markowitz Model (Mean-Variance Portfolio Theory):

Differences:

Focus:

Markowitz Model focuses on individual assets and their correlations in a portfolio. It emphasizes the trade-off between risk and return for individual securities.

Index Model: It is concerned with the relationship between individual stock returns and the return of the market index.

Objective:

Markowitz Model: Aims to maximize portfolio expected return for a given level of risk or minimize portfolio risk for a given level of expected return.

Index Model: Seeks to understand the factors that explain the return of an individual stock about the return of the overall market index.

Mathematics:

Markowitz Model: Involves calculating the expected returns, variances, and covariances of individual assets to construct an efficient frontier.

Index Model: Employs regression analysis to identify the relationship between individual stock returns and market index returns.

Common Places:

Risk Assessment:

Both models are concerned with assessing and managing risk in investment portfolios.

Diversification:

Both models emphasize the importance of diversification in reducing portfolio risk. Markowitz's model quantifies this by looking at the correlation between assets, while the index model considers how individual stock returns move with the market index.

Portfolio Construction:

Investors can use insights from both models to construct well-diversified portfolios. Markowitz's model helps select assets with low correlations, while the index model helps understand how individual stocks behave concerning the broader market movement.

Statistical Analysis:

Both models use statistical techniques. Markowitz's model uses variance and covariance calculations, whereas the index model uses regression analysis to quantify the relationship between individual stock returns and the market index.