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# RESEARCH PAPER FOR USING MARKOWITZ MODEL AND INDEX MODEL UNDER REALISTIC ADDITIONAL CONSTRAINTS

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## 1. Abstract

The Markowitz and index models are widely used in portfolio optimization to achieve optimal asset allocation and maximize returns while controlling risk. However, in practical investment scenarios, additional constraints often need to be considered to ensure the feasibility and effectiveness of the portfolio strategy. This research focuses on incorporating realistic additional constraints into the Markowitz and index models for portfolio optimization. The study utilizes historical financial data and statistical techniques to estimate expected returns, variances, and covariance matrices. The Markowitz model is modified to incorporate the additional constraints, allowing for more realistic and accurate portfolio optimization outcomes. The index model is also adapted to consider the impact of these constraints on the construction and performance of passive investment portfolios.

**Keywords:** Markowitz model, index model

## 2. Introduction

In this study, we utilize the most recent 20 years of historical daily total return data for the S&P 500 index (represented by the ticker symbol "SPX") and ten U.S. stocks as our data set. Our objective is to determine the appropriate optimization inputs for both the Markowitz Model ("MM") and the Index Model ("IM") and subsequently identify the permissible regions of portfolios, including the efficient frontier, minimal risk portfolio, optimal portfolio, and minimal return portfolios frontier, considering five additional constraints. To ensure accurate results, we first aggregate the daily data into monthly observations, which helps mitigate the impact of non-Gaussian effects. Next, we calculate the correct optimization inputs for the MM and I.M. models. Finally, by applying the optimization techniques of the MM and I.M. models, we identify the areas where portfolios satisfying the five additional constraints are permissible.

### 2.1 The Markowitz Model

The Markowitz model, or Modern Portfolio Theory (MPT), is a fundamental concept in finance and investment. Developed by economist Harry Markowitz in 1952, the model revolutionized portfolio management by introducing a quantitative approach to asset allocation. (Beste, A. & Leventhal, D. & Williams, J. & Lu, Q. ,2002).The primary objective of the Markowitz model is to construct an optimal portfolio that maximizes expected returns for a given level of risk or minimizes risk for a given level of expected returns. It is based on the premise that investors are risk-averse and seek to achieve the highest possible return while minimizing the associated uncertainty. The model utilizes statistical techniques to estimate individual assets' expected returns, variances, and covariances. By considering these inputs, along with the investor's risk tolerance and desired return objectives, the Markowitz model generates an efficient frontier, which represents the set of portfolios that offer the highest return for a given level of risk or the lowest risk for a given level of return.

### 2.2 The Index Model

The index model, also known as the market model or the single-factor model, is a widely used tool in finance for measuring and analyzing the relationship between the returns of individual securities and the overall market (Hristache, M., Juditsky, A., & Spokoiny, V., 2001). It provides insights into the systematic risk and performance of specific assets within broader market movements. The key idea behind the index model is to decompose the return on an individual security into two components: systematic risk and idiosyncratic risk. Systematic risk refers to the portion of the security's return attributable to common market factors, such as changes in interest rates, economic indicators, or overall market sentiment. On the other hand, idiosyncratic risk represents the unique or company-specific factors that affect the security's return and are unrelated to the broader market. The index model employs regression analysis to estimate the relationship between individual securities' returns and the market index's returns. By regressing the historical returns of security against the returns of the market index, the model quantifies the security's beta, which measures its sensitivity to systematic risk. A beta greater than one indicates that the security tends to move more than the market, whereas a beta less than 1 suggests it moves less. One of the primary applications of the index model is portfolio management. By incorporating the estimated betas of individual securities, investors can construct diversified portfolios that target specific levels of systematic risk exposure. This approach enables investors to balance risk and return by allocating assets according to risk preferences and desired market exposures.

For the Index Model, in terms of individual stocks:

$$Cov\left(r_i, r_j\right) = E\left[r_i - E\left[r_i\right]\right)\left(r_j - E\left[r_j\right]\right) \tag{2.1}$$

$$\sigma_i^2 = Cov\left(r_i, r_j\right) \quad i = j \tag{2.2}$$

In addition,

$$Corr\left(r_i, r_j\right) = \frac{Cov\left(r_i, r_j\right)}{\sigma_i \sigma_j} \tag{2.3}$$

The return of portfolios:

$$E\left(R_p\right) = \sum_{i=1}^{N} w_i E\left(r_i\right) \tag{2.4}$$

The risk of portfolios:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j Cov\left(r_i, r_j\right) \tag{2.5}$$

# 3. Research Outline

## 3.1 Background of the companies

I collected the daily returns of ten stocks, the S&P 500 index, and the one-month federal funds rate over the last twenty years. Data from Yahoo! Finance.

| # | Group #3 | Full Name | Sector (Yahoo!finance) |
|---|---|---|---|
| 1 | NVDA | NVIDIA Corporation | Technology |
| 2 | CSCO | Cisco Systems, Inc. | Technology |
| 3 | INTC | Intel Corporation | Technology |
| 4 | GS | The Goldman Sachs Group, Inc. | Financial Services |
| 5 | USB | U.S. Bancorp | Financial Services |
| 6 | TD CN | The Toronto-Dominion Bank | Financial Services |
| 7 | ALL | The Allstate Corporation | Financial Services |
| 8 | PG | The Procter & Gamble Company | Consumer Defensive |
| 9 | JNJ | Johnson & Johnson | Healthcare |
| 10 | CL | Colgate-Palmolive Company | Consumer Defensive |

a) NVDA

NVIDIA Corporation is a multinational technology company headquartered in Santa Clara, California, USA. The company specializes in designing and developing graphics processing units (GPUs) and system-on-a-chip units (SoCs) for use in various industries, including gaming, professional visualization, data centers, and automotive.

b) CSCO

Cisco Systems, Inc. is a multinational technology company headquartered in San Jose, California, USA. Established in 1984, Cisco is a leading networking hardware, software, and services provider for a wide range of customers, including businesses, government agencies, and service providers.

c) INTC

Intel Corporation, commonly known as Intel, is a multinational technology company based in Santa Clara, California, USA. Founded in 1968, Intel is one of the world's largest and most prominent semiconductor chip manufacturers.

d) GS

Goldman Sachs Group, Inc. is a multinational investment bank and financial services company headquartered in New York City, USA. Founded in 1869, Goldman Sachs is a globally prestigious and influential financial institution.

e) USB

U.S. Bancorp is a diversified financial services company based in the United States. It is one of the largest commercial banks in the country and has its headquarters in Minneapolis, Minnesota. U.S. Bancorp operates through its main subsidiary, U.S. Bank National Association, which offers a wide range of banking and financial services.

f) T.D. C.N.

The Toronto-Dominion Bank, commonly known as T.D. Bank is one of Canada's largest and most prominent financial institutions. Headquartered in Toronto, Ontario, T.D. Bank is a multinational bank with a significant presence in Canada and the United States.

g) ALL

The Allstate Corporation is one of the largest insurance companies in the United States, providing customers a wide range of insurance products and financial services nationwide. The company is headquartered in Northbrook, Illinois, and was founded in 1931.

h) P.G.

The Procter & Gamble Company is a multinational consumer goods corporation headquartered in Cincinnati, Ohio. The company was founded in 1837, and today, it is one of the world's largest and most successful consumer goods companies.

i) JNJ

Johnson & Johnson is a multinational corporation specializing in healthcare and consumer goods. Headquartered in New Brunswick, New Jersey, Johnson & Johnson is one of the largest and most well-known healthcare companies globally.

j) CL
Colgate-Palmolive is a multinational consumer goods company specializing in oral care, personal care, home care, and pet nutrition products. Colgate-Palmolive is headquartered in New York City and is recognized as one of the world's largest and most well-known consumer goods companies.

## 3.2 Constraints

3.2.1 This additional optimization constraint is designed to simulate Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity:

$$\sum_{i=1}^{11} |w_i| \leqslant 2;$$

**3.2.2 This additional optimization constraint is designed to simulate some arbitrary "box" constraints on weights, which the client may provide:**

$$|w_i| \leqslant 1, for \forall i;$$

**3.2.3 A "free" problem, without any additional optimization constraints, to illustrate how the area of permissible portfolios in general and the efficient frontier in particular look like if you have no constraints;**

**3.2.4 This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions:**

$$w_i \geqslant 0, for \forall i$$

**3.2.5 Lastly, we would like to see if the inclusion of the broad index into our portfolio has a positive or negative effect; for that, we would like to consider an additional optimization constraint:**

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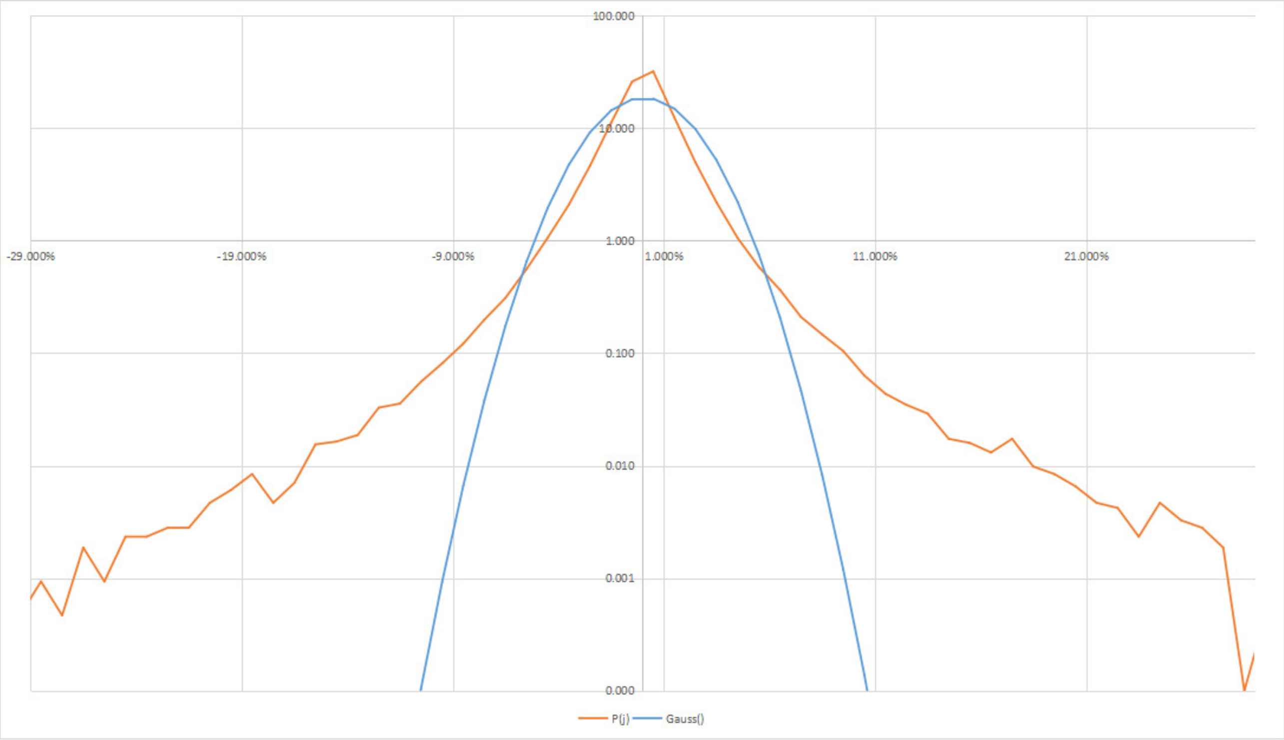
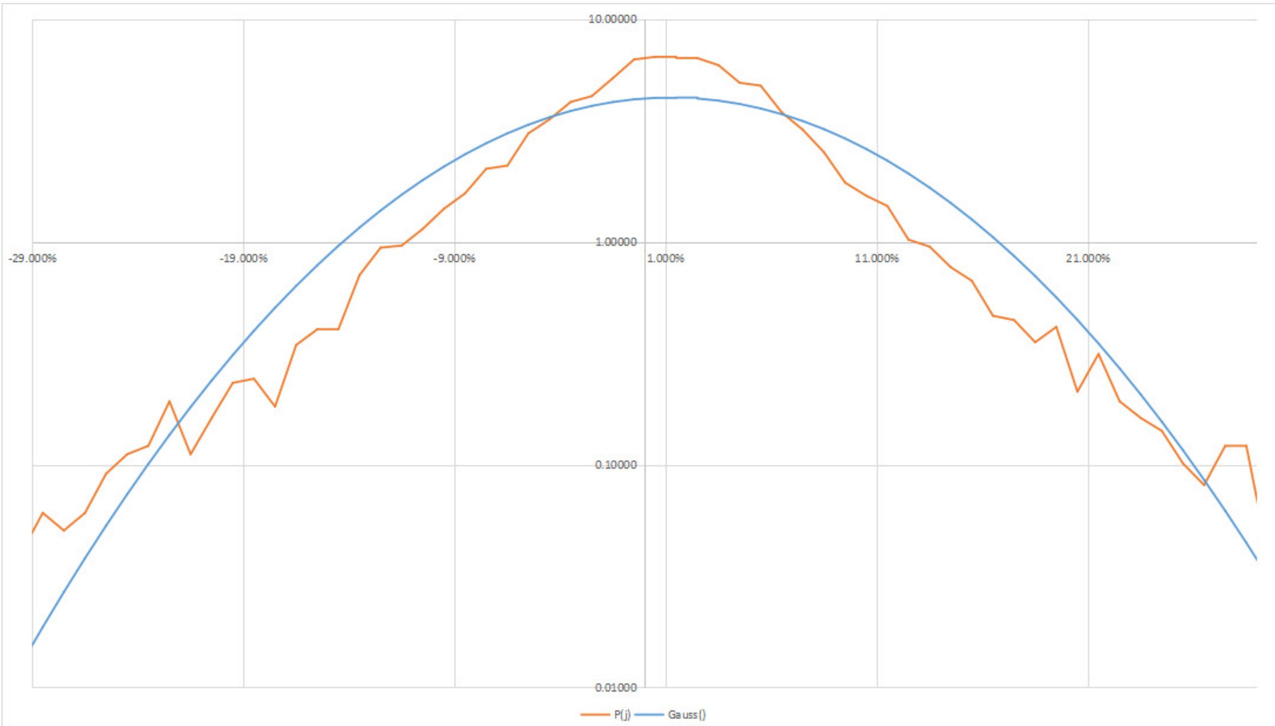
# 4. Data Analysis

If we take daily data into the calculation, it is obvious that the distribution is strongly non-Gaussian.




If we use the daily data to estimate the monthly data, we can find that the monthly data is a lot closer to the Gaussian distribution than the daily data.

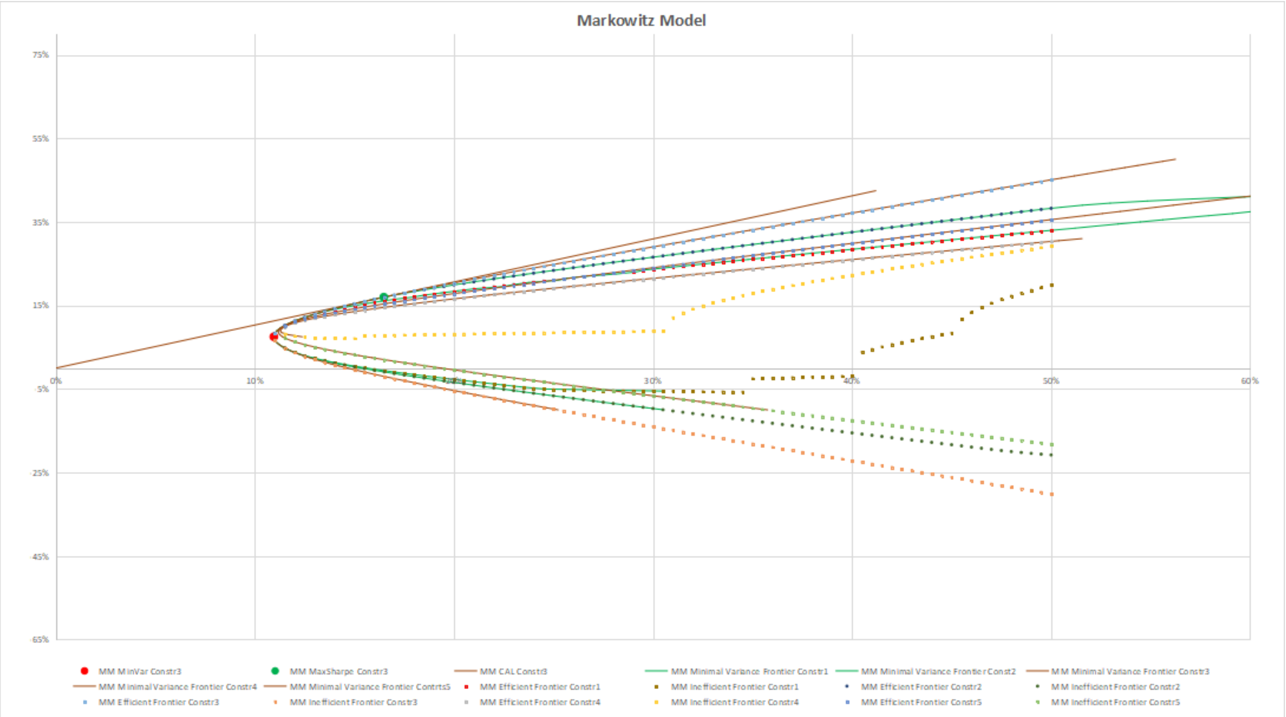


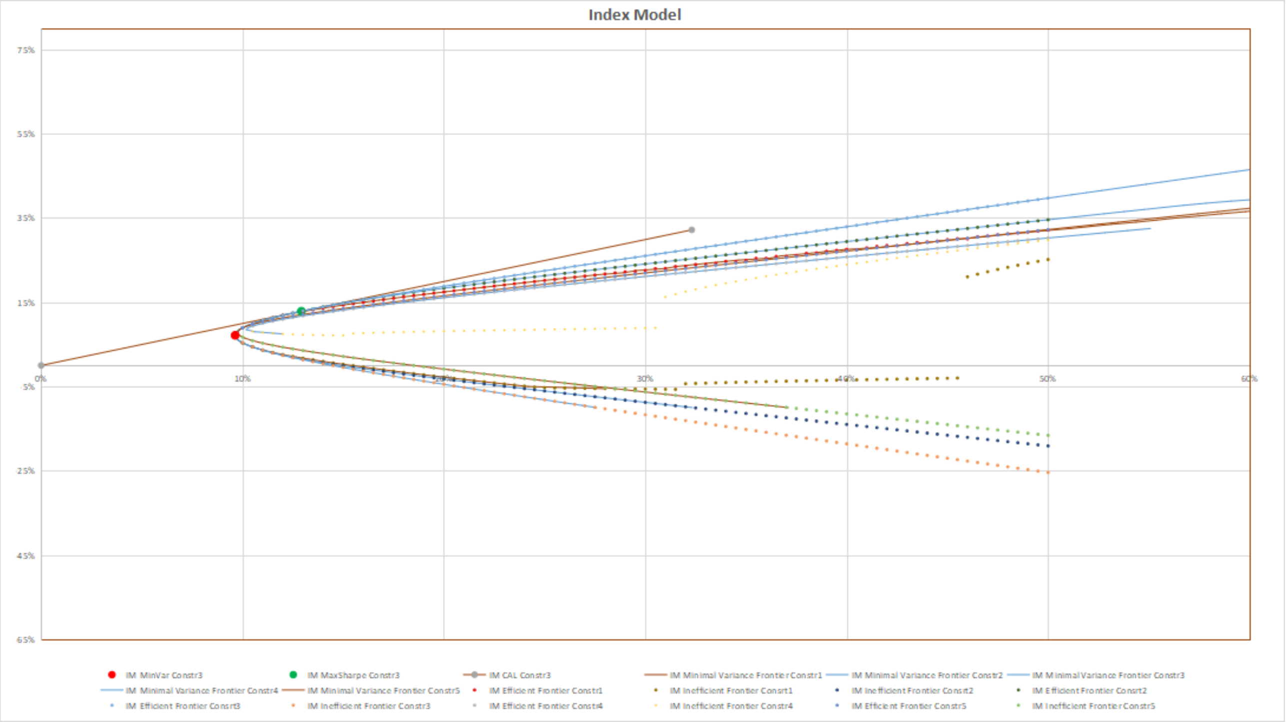

From the monthly data, we estimate the annual data, which is:

| | SPX | NVDA | CSCO | INTC | GS | USB | TD CN | ALL | PG | JNJ | CL |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Annual Average Return | 7.5% | 32.8% | 9.7% | 8.9% | 10.8% | 9.9% | 11.0% | 10.1% | 9.4% | 8.5% | 7.1% |
| Annual StDev | 14.9% | 55.8% | 30.8% | 30.5% | 29.6% | 23.7% | 18.1% | 24.9% | 14.6% | 14.8% | 15.3% |
| beta | 1.00 | 1.98 | 1.32 | 1.19 | 1.41 | 0.97 | 0.79 | 1.06 | 0.41 | 0.54 | 0.45 |
| alpha | 0.00 | 0.18 | 0.00 | 0.00 | 0.00 | 0.03 | 0.05 | 0.02 | 0.06 | 0.04 | 0.04 |
| residual Stdev | 0.0% | 47.4% | 23.8% | 24.9% | 20.9% | 18.8% | 13.9% | 19.3% | 13.3% | 12.4% | 13.8% |

Correlations

| | SPX | NVDA | CSCO | INTC | GS | USB | TD CN | ALL | PG | JNJ | CL |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SPX | 100.0% | 52.7% | 63.7% | 57.8% | 70.8% | 60.9% | 64.5% | 63.0% | 41.2% | 54.2% | 44.0% |
| NVDA | 52.7% | 100.0% | 48.7% | 52.4% | 34.3% | 16.0% | 33.8% | 15.7% | 6.0% | 16.5% | 6.9% |
| CSCO | 63.7% | 48.7% | 100.0% | 61.4% | 48.7% | 32.8% | 41.0% | 29.7% | 22.0% | 23.9% | 16.5% |
| INTC | 57.8% | 52.4% | 61.4% | 100.0% | 41.1% | 28.0% | 41.2% | 28.6% | 13.6% | 32.5% | 11.0% |
| GS | 70.8% | 34.3% | 48.7% | 41.1% | 100.0% | 47.2% | 49.4% | 41.7% | 17.3% | 29.6% | 20.3% |
| USB | 60.9% | 16.0% | 32.8% | 28.0% | 47.2% | 100.0% | 53.9% | 54.0% | 33.6% | 23.4% | 21.8% |
| TD CN | 64.5% | 33.8% | 41.0% | 41.2% | 49.4% | 53.9% | 100.0% | 41.7% | 23.1% | 27.3% | 21.2% |
| ALL | 63.0% | 15.7% | 29.7% | 28.6% | 41.7% | 54.0% | 41.7% | 100.0% | 34.6% | 45.2% | 40.7% |
| PG | 41.2% | 6.0% | 22.0% | 13.6% | 17.3% | 33.6% | 23.1% | 34.6% | 100.0% | 49.4% | 48.3% |
| JNJ | 54.2% | 16.5% | 23.9% | 32.5% | 29.6% | 23.4% | 27.3% | 45.2% | 49.4% | 100.0% | 52.7% |
| CL | 44.0% | 6.9% | 16.5% | 11.0% | 20.3% | 21.8% | 21.2% | 40.7% | 48.3% | 52.7% | 100.0% |


Markowitz Model
75%
55%
35%
15%
-5%
-25%
-45%
-65%
0%
10%
20%
30%
40%
50%
60%
MM MinVar Constr3
MM MaxSharpe Constr3
MM CAL Constr3
MM Minimal Variance Frontier Constr1
MM Minimal Variance Frontier Const2
MM Minimal Variance Frontier Constr3
MM Minimal Variance Frontier Constr4
MM Minimal Variance Frontier Contrts5
MM Efficient Frontier Constr1
MM Inefficient Frontier Constr1
MM Efficient Frontier Constr2
MM Inefficient Frontier Constr2
MM Efficient Frontier Constr3
MM Inefficient Frontier Constr3
MM Efficient Frontier Constr4
MM Inefficient Frontier Constr4
MM Efficient Frontier Constr5
MM Inefficient Frontier Constr5


Index Model
75%
55%
35%
15%
-5%
-25%
-45%
-65%
0%
10%
20%
30%
40%
50%
60%
IM MinVar Constr3
IM MaxSharpe Constr3
IM CAL Constr3
IM Minimal Variance Frontier Constr1
IM Minimal Variance Frontier Constr2
IM Minimal Variance Frontier Constr3
IM Minimal Variance Frontier Constr4
IM Minimal Variance Frontier Constr5
IM Efficient Frontier Constr1
IM Inefficient Frontier Consrt1
IM Inefficient Frontier Consrt2
IM Efficient Frontier Consrt2
IM Efficient Frontier Consrt3
IM Inefficient Frontier Constr3
IM Efficient Frontier Constr4
IM Inefficient Frontier Constr4
IM Efficient Frontier Constr5
IM Inefficient Frontier Constr5

## 4.1 Constraint 1

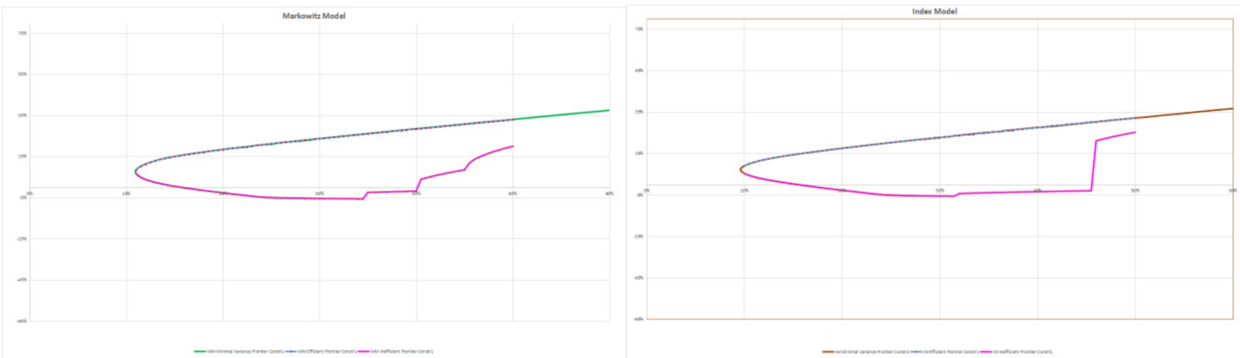



**Figure 4.1**

The Efficient Frontier, Inefficient Frontier, and Minimal Variance Frontier in Figure 4.1 are almost identical for both the Markowitz and Index models, and the difference is negligible. The situation undergoes a significant change when the standard deviation of the Inefficient Frontier exceeds 34.5%. In this case, the Inefficient Frontier for the I.M. (Index model) experiences a sharp increase when the standard deviation of the Inefficient Frontier equals 34.5%, 40%, and 45%. On the other hand, the Inefficient Frontier for the MM (Markowitz model) surpasses the I.M. Inefficient Frontier when the standard deviation reaches 46% and remains relatively stable.

## 4.2 Constraint 2

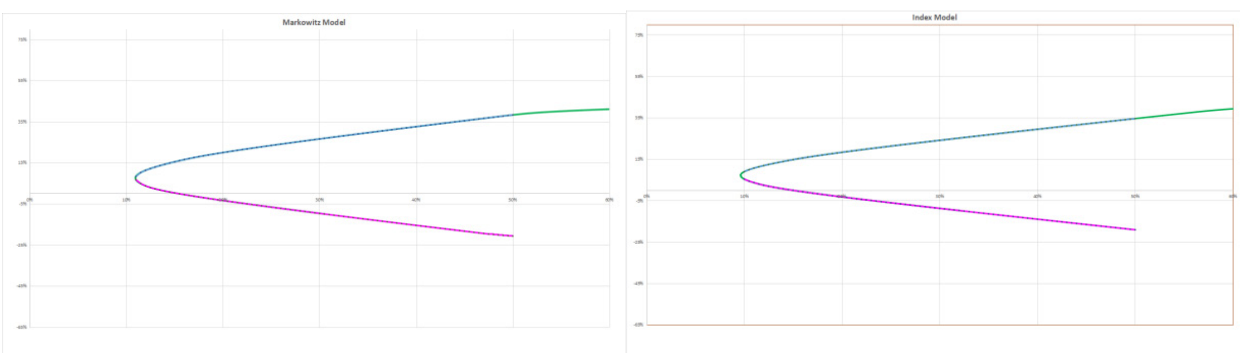



**Figure 4.2**

In Figure 4.2, The Efficient Frontier, Inefficient Frontier, and Minimal Variance Frontier for the Markowitz and Index models exhibit similar characteristics. Within the Efficient Frontier, the I.M. efficient frontier slightly surpasses the MM efficient frontier in the range when the standard deviation is lower than 13%. With the standard deviation rising, the MM efficient frontier is slightly higher than the I.M. efficient frontier. In the Inefficient Frontier, when the standard deviation is lower than 18%, the MM inefficient frontier is higher than the I.M. inefficient frontier. On the contrary, the MM inefficient frontier is lower than the I.M. inefficient frontier in the range of 18% to 50% of the standard deviation.

## 4.3 Constraint 3

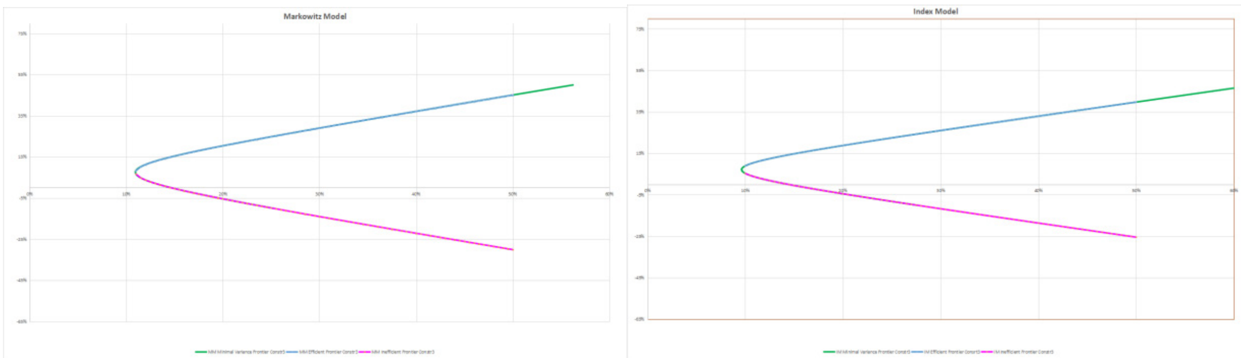



**Figure 4.3**

The shapes of the Markowitz and Index model in Figure 4.3 are quite similar to the shapes in Figure 4.2. From an overall shape perspective, The Index model exhibits a narrower back-end opening, which means that with the standard deviation increase, the MM efficient frontier gets higher than the I.M. efficient frontier, and the MM inefficient frontier gets lower than the I.M. inefficient frontier. Specifically, 13% of the standard deviation for the efficient frontier and 15.5% of the standard deviation for the inefficient frontier refer to two crossover points.

## 4.4 Constraint 4

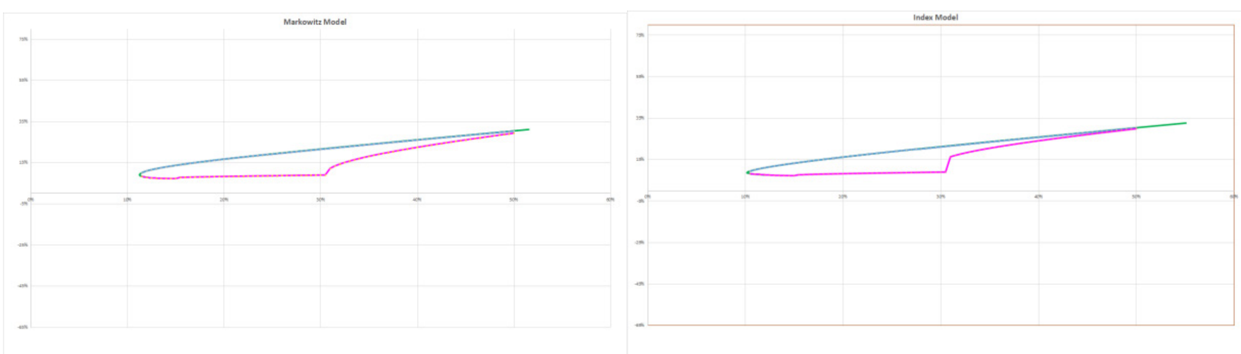



**Figure 4.4**

In Figure 4.4, the Markowitz model efficient frontier is a little higher than the Index model efficient frontier in the 12.5% to 50% of standard deviation. When the standard deviation is lower than 31%, the inefficient frontier for Markowitz and Index models is Essentially no difference. However, at the standard deviation of 31%, both the MM and the I.M. inefficient frontier experience a surge, and compared with the MM, the I.M. inefficient frontier has a larger increment. On the contrary, the MM inefficient frontier has a larger slope of 31.5% to 50% of standard deviation. With the standard deviation increase, compared with the previous constraints, the inefficient frontier for MM and I.M. approach the efficient frontier and are about to overlap.

### 4.5 Constraint 5

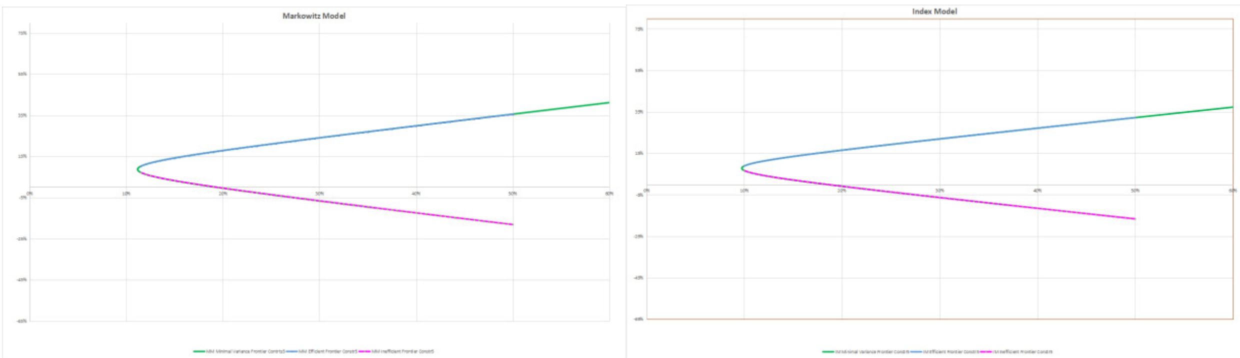



**Figure 4.5**

The Efficient Frontier, Inefficient Frontier, and Minimal Variance Frontier for the Markowitz and Index models in Figure 4.5 are very similar to those in Constraint 2 and Constraint 3. When the standard deviation exceeds 12.5%, the MM efficient frontier surpasses the I.M. efficient frontier. To compare the MM and I.M. inefficient frontier, we can easily find that the I.M. inefficient frontier is lower than the MM inefficient frontier until the standard deviation increases to 25.5%. Similarly, The Index model exhibits a narrower back-end opening than the Markowitz model as it performs in constraints 2 and 3.

## 5. Conclusion

For the minimal variance portfolio, the results of the Markowitz model show higher standard deviation and expected return compared with the results of the Index model. However, the Sharpe ratio of the minimal variance portfolio for MM is lower than the portfolio Sharpe ratio using I.M. in every constraint. As for the efficient risky portfolio, the Markowitz model still has a higher expected return with a higher standard deviation. Unlike the situation above, there is a difference in this particular context. This time, MM shows a higher Sharpe ratio, which suggests that we can get a higher return per unit of risk.

| MM (Constr1): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 38.37% | -2.97% | -2.89% | 1.33% | -5.90% | -0.30% | 19.41% | -11.48% | 25.93% | 18.83% | 19.67% | 7.51% | 10.95% | 0.685 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -42.74% | 15.75% | -1.15% | -6.11% | 3.25% | 6.48% | 35.29% | 1.07% | 45.71% | 30.00% | 12.45% | 14.01% | 13.95% | 1.004 | 2.5 | 35.0% | 34.9% |
| MM (Constr2): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 38.37% | -2.97% | -2.89% | 1.33% | -5.90% | -0.30% | 19.41% | -11.48% | 25.93% | 18.83% | 19.67% | 7.51% | 10.95% | 0.685 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -100.00% | 21.50% | 0.31% | -8.15% | 11.46% | 12.25% | 44.92% | 6.87% | 52.33% | 41.02% | 17.48% | 16.56% | 16.06% | 1.031 | 2.5 | 41.4% | 40.2% |
| MM (Constr3): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 38.37% | -2.97% | -2.89% | 1.33% | -5.90% | -0.30% | 19.41% | -11.48% | 25.93% | 18.83% | 19.67% | 7.51% | 10.95% | 0.685 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -109.97% | 22.46% | 0.89% | -8.19% | 12.73% | 13.21% | 46.46% | 7.90% | 53.50% | 42.72% | 18.30% | 16.99% | 16.48% | 1.031 | 2.5 | 42.5% | 41.2% |
| MM (Constr4): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 9.49% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 19.85% | 0.00% | 28.91% | 20.62% | 21.13% | 8.88% | 11.27% | 0.788 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 10.95% | 0.00% | 0.00% | 0.00% | 0.00% | 23.73% | 0.00% | 42.56% | 16.17% | 6.60% | 12.06% | 13.12% | 0.919 | 2.5 | 30.1% | 32.8% |
| MM (Constr5): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 0.00% | -0.97% | 0.08% | 2.51% | -0.99% | 3.50% | 24.70% | -8.17% | 28.91% | 25.58% | 24.85% | 8.71% | 11.18% | 0.779 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 14.93% | -6.85% | -10.14% | -1.30% | 2.43% | 30.65% | -2.27% | 43.31% | 23.61% | 5.64% | 13.06% | 13.69% | 0.954 | 2.5 | 32.6% | 34.2% |

| IM (Constr1): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 25.62% | -4.04% | -5.27% | -2.81% | -8.73% | 0.76% | 10.28% | -1.40% | 31.27% | 27.68% | 26.64% | 7.15% | 9.63% | 0.742 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -47.62% | 8.88% | -1.24% | -0.49% | -0.64% | 6.67% | 29.55% | 4.57% | 43.95% | 33.35% | 23.01% | 12.07% | 12.18% | 0.990 | 2.5 | 30.2% | 30.5% |
| IM (Constr2): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 25.62% | -4.04% | -5.27% | -2.81% | -8.73% | 0.76% | 10.28% | -1.40% | 31.27% | 27.67% | 26.64% | 7.15% | 9.63% | 0.742 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -70.16% | 10.32% | -0.57% | -0.11% | 0.57% | 9.39% | 34.25% | 7.35% | 46.90% | 36.94% | 25.11% | 12.87% | 12.92% | 0.996 | 2.5 | 32.2% | 32.3% |
| IM (Constr3): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 25.62% | -4.04% | -5.27% | -2.81% | -8.73% | 0.76% | 10.28% | -1.40% | 31.27% | 27.67% | 26.64% | 7.15% | 9.63% | 0.742 | CAL: | 0.0% | 0.0% |
| MaxSharpe | -70.16% | 10.32% | -0.57% | -0.11% | 0.57% | 9.39% | 34.25% | 7.35% | 46.90% | 36.94% | 25.11% | 12.87% | 12.92% | 0.996 | 2.5 | 32.2% | 32.3% |
| IM (Constr4): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 9.21% | 0.00% | 33.55% | 28.89% | 28.35% | 8.64% | 10.16% | 0.850 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 6.74% | 0.00% | 0.00% | 0.00% | 0.00% | 17.75% | 0.00% | 37.34% | 22.75% | 15.41% | 10.71% | 11.72% | 0.914 | 2.5 | 26.8% | 29.3% |
| IM (Constr5): | SPX | AMZN | AAPL | CTXS | JPM | BRK/A | PGR | UPS | FDX | JBHT | LSTR | Return | StDev | Sharpe | | | |
| MinVar | 0.00% | -3.35% | -3.28% | -1.14% | -6.02% | 3.27% | 14.25% | 1.13% | 34.13% | 31.54% | 29.47% | 7.82% | 9.75% | 0.802 | CAL: | 0.0% | 0.0% |
| MaxSharpe | 0.00% | 7.68% | -5.84% | -4.45% | -6.81% | 2.56% | 22.82% | 0.47% | 38.76% | 26.69% | 18.11% | 10.84% | 11.48% | 0.944 | 2.5 | 27.1% | 28.7% |

This article introduces modern portfolio theory and its two data models, namely the Markowitz model and the Index Model. It presents the formulas for calculating a portfolio's return and risk. Following the explanation of these models, the article discusses the data sources, specifically ten stocks, and details their daily returns and consolidated weekday returns over the past 20 years. Monthly returns are adjusted for, and calculations are made to determine each stock's probability distribution, kurtosis, skewness, and excess return, which helps assess if the data follows a normal distribution for further analysis. The correlation coefficient between each stock and the index is then determined to measure systematic and stock-specific risk in the capital market. Four constraints are mentioned in the article, each corresponding to a different control, enhancing the final calculations' relevance. By applying these two models, the article calculates the asset portfolio's return, variance, and Sharpe ratio. Since investors typically aim to reduce risk while maintaining the same level of return, an efficient and convenient asset allocation strategy seeks to maximize returns. Finally, the article compares the results of the Index Model and Markowitz model under the four constraints, providing a wide range of investment alternatives for individuals to evaluate based on specific situations.

## References

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