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Applying and analyzing the Model Portfolio Theory (MPT) by using Markowitz Model and index Model

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ABSTRACT:

In this study, I apply the Markovitz Model("MM") and Index Model("IM"). These Portfolio Optimization models estimate and optimize U.S. equity portfolios with some realistic additional constraints.

I was given a recent 20 years of historical daily total return data for ten stocks, which belong in groups to three different sectors (according to Yahoo! Finance), one (S&P 500) equity index (a total of eleven risky assets), and a proxy for risk-free rate (1-month Fed Funds rate). I aggregate the daily data to the monthly observations, and based on those monthly observations, I calculate all proper optimization inputs for the full Markowitz Model ("MM") alongside the Index Model ("IM"). We use the optimization inputs ("IM") and ("MM") to explore some additional constraints.

Keywords: Markovitz Model, Index Model, Optimal portfolio, additional constraints

1. Introduction:

Modern Portfolio Theory is a financial framework developed by Harry Markowitz in the 1950s, earning him a Nobel Prize. MPT aims to maximize returns while minimizing risk by diversifying investments across different asset classes.

The main idea behind MPT is that an investor can reduce portfolio risk by holding a diversified portfolio of assets. According to MPT, certain assets may perform better under different market conditions.

The overall portfolio risk can be reduced by spreading investments across multiple asset classes.

MPT was developed in response to the traditional investment approach of investing primarily in stocks and bonds. Markowitz argued that this approach did not adequately account for the benefits of diversification. He believed investors could achieve better returns with less risk through a more diversified approach.

The mathematical framework operating in modern portfolio theory is a practical strategy in investment selection using diversification as a fundamental principle.

The formula aims to maximize the expected return at a given specific level of risk.

2. Theoretical Model

2.1 Single-index Model

The single-index model is a simple asset pricing model to measure a stock's risk and return. The model has been developed by William Sharpe in 1913.

Two assumptions in William Sharpe's single Index model.

1) The risk of securities is divided into systematic risk and non-systematic risk, and the factors have no impact on the non-systematic risk;

2) The unsystematic risk of one security does not affect the unsystematic risk of other securities, and the returns of both securities are only correlated by the combined reaction of factors.

The above two assumptions imply that Cov(Rm,εi)=0; Cov (εi, εj)=0; This simplifies the calculations to a great extent.

Relates returns on each security to the returns on a common index, such as the S&P 500 Stock Index

Expressed by the following equation

The **expected return of the IM model portfolio** is

The **Standard deviation of the Portfolio** is:

r_i: the expected rate of return of Asset i

w_i: the proportion of asset i in the portfolio

n: the number of total assets,

: the risk factor of asset i

: systematic risk

: unsystematic risk

2.2The Full Markowitz Model

Markowitz's model is an optimal financial investment strategy to maximize the expected return for an investor while maintaining a desired level of risk.

The Markowitz model of risk-return optimization is a portfolio selection model that derives a set of weights for an investment portfolio that minimizes the total variance of returns, subject to an initial capital constraint.

Dr Harry M. Markowitz was the first to develop the modern portfolio analysis model. He developed it in the 1950s. Markowitz started with the average investor's new idea of risk aversion and their desire to maximize the expected return with the least risk. He provided a theoretical framework to analyze the risk and returns and their inter-relationship. His framework helps in the efficient choice portfolio. An efficient portfolio is a combination of securities that provide the highest return for a given level of risk and the lowest risk for a given level of return.

Markowitz's model is called the "Full covariance model" because, with the help of this model, the investor can find out the efficient set of the portfolio by finding out the trade-off between risk and return, between the limit of zero and infinity.

Assumptions of Markowitz's Risk and Return Theory.

The Markowitz model theory of risk and return optimization is based on the following assumptions:

1)Investors are rational and risk-averse: The theory assumes that investors are rational and seek to maximize their returns while minimizing their risks. Investors will always prefer a portfolio with higher expected returns and lower risk.

2)Asset returns are normally distributed: Markowitz's theory assumes asset returns follow a normal distribution. This means that the returns are symmetrically distributed around the mean, and the majority of returns fall within a certain range.

3)Correlation between assets is known: The theory assumes that the correlation between assets can be accurately measured. This allows investors to construct portfolios that are well-diversified and minimize risk.

4) No transaction costs: Markowitz's theory assumes no transaction costs are associated with buying and selling assets. This allows investors to freely move in and out of different assets without incurring additional costs.

5) Investment horizon is long-term: The theory assumes that investors have a long-term investment horizon and are willing to hold their portfolios for an extended period. This allows investors to benefit from the long-term growth potential of their investments.

The **expected return of the MM model portfolio** is:

$$R_p = \sum_{i=1}^{n} w_i r_i$$

The **standard deviation of the portfolio** is:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j cov(R_i, R_j)}$$

r_i : the expected return on asset i

w_i : represents the proportion of assets i in the portfolio

i : the number of total assets

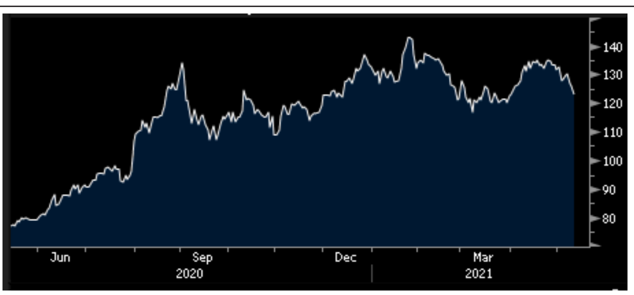
$cov(R_i, R_j)$ represents the covariance between the return on asset i and the return on asset j

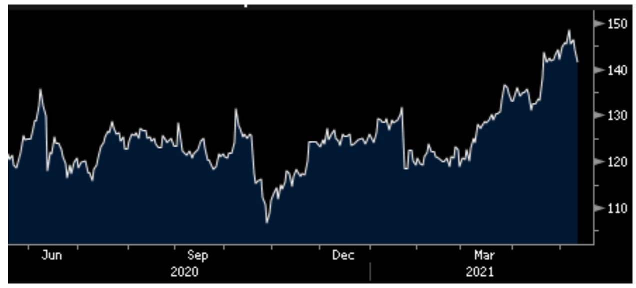
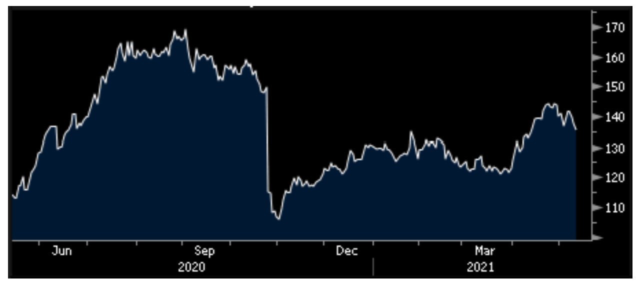
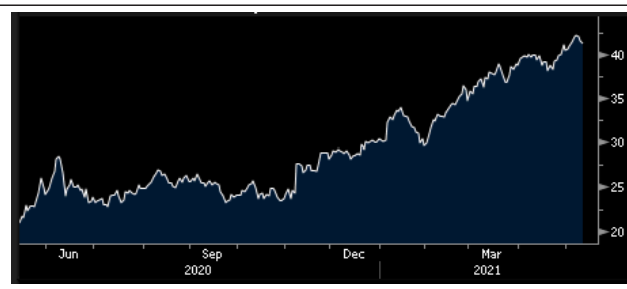
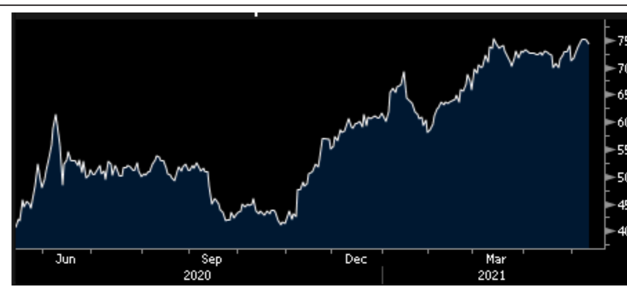
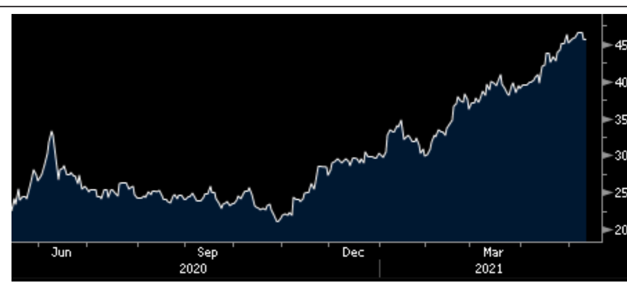
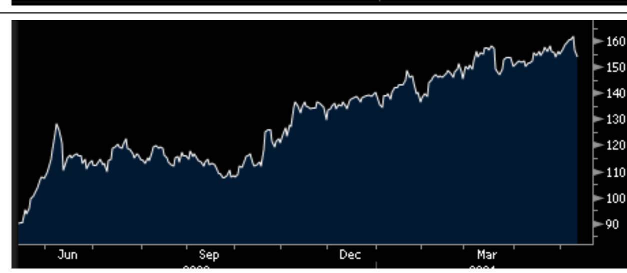
3. ASSUMPTION TEST

3.1 Data Description

I select ten stocks from three different equity sectors (according to Yahoo Finance): technology, financial services, and industrials, to validate the model theory and use the S&P 500 as a market index (11 risky assets in total) and a proxy for a risk-free rate (1 month's federal funds rate). Using Bloomberg Professional, we obtained the daily data of these stocks from May 11, 2001, to May 12, 2021, over the last 20 years. We further processed the data to include only five working days of daily data per each week and produced the corresponding monthly data. Tables 1 and 2 introduce the ten stocks. (Data from Yahoo! Finance)

Table 1 Description of ten stocks

Stock code	Stock name	Stock Sector	Price Chart
ADBE	Adobe Inc.	Technology	

IBM	International Business Machines Corporation	Technology	
SAP	SAP SE	Technology	
BAC	Bank of America Corporation	Financial Services	
C	Citigroup Inc.	Financial Services	
WFC	Wells Fargo & Company	Financial Services	
TRV	The Travelers Companies, Inc.	Financial Services	

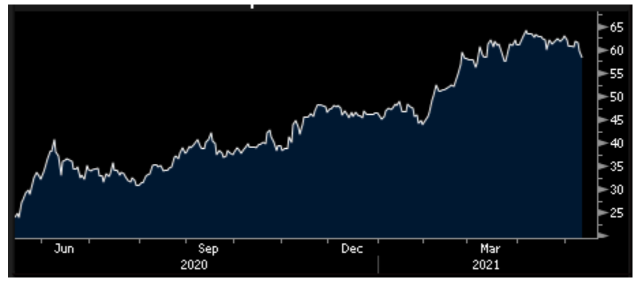
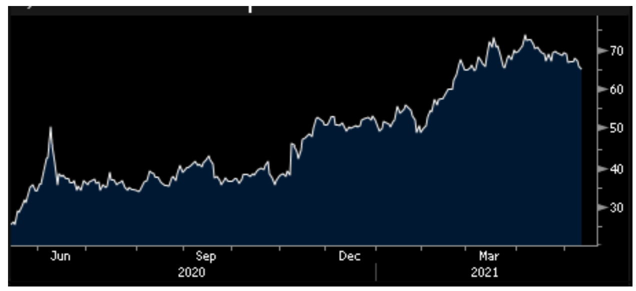
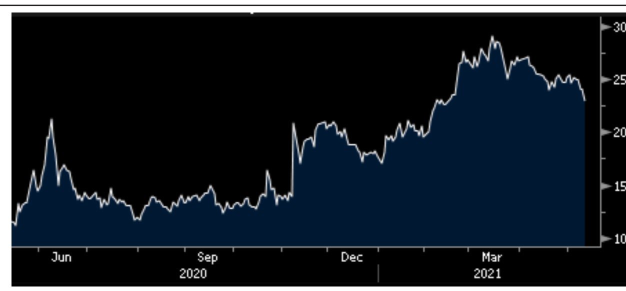
LUV	Southwest Airlines Co.	Industrials	
ALK	Alaska Air Group, Inc.	Industrials	
HA	Hawaiian Holdings, Inc.	Industrials	

Table 2

#	Group #1	Full Name	Sector (Yahoo!finance)
1	ADBE	Adobe Inc.	Technology
2	IBM	International Business Machines Corporation	Technology
3	SAP	SAP SE	Technology
4	BAC	Bank of America Corporation	Financial Services
5	C	Citigroup Inc.	Financial Services
6	WFC	Wells Fargo & Company	Financial Services
7	TRV	The Travelers Companies, Inc.	Financial Services
8	LUV	Southwest Airlines Co.	Industrials
9	ALK	Alaska Air Group, Inc.	Industrials
10	HA	Hawaiian Holdings, Inc.	Industrials

3.2 Constraints

We tested four conditions when calculating the Efficient Frontier, Inefficient Frontier, and Minimal Variance Frontier for the Markowitz Model and Index Model, respectively. Adding the constraints makes the final calculation results closer to real-life situations, such as governmental restrictions.

1. The first condition is that the sum of the absolute values of the weights of all individual stocks cannot be greater than 2. This optimization constraint is designed to mimic FINRA's Regulation T. This constraint allows a broker-dealer to sell a stock to a broker-dealer if the broker-dealer's weight is not greater than 2. Regulation T allows a broker-dealer to allow its customers to own positions greater than or equal to 50% of the position funded by account equity.

2. The second requirement is that the absolute value of each stock needs to be less than or equal to 1. Such optimization constraints are designed to simulate the weighting constraints still in place, which the user's hedge fund may provide.

3. The third condition is that there are no constraints. This is to show what the efficient frontier looks like.

4. The last condition tested was that the weight of each stock needed to be rainy or equal to 0. This optimization constraint simulates that open-end mutual funds in the United States cannot have short positions.

We added an extra step to the calculation to calculate the absolute value in Excel and not let the computer report an error. It is the square root of the value of the square of the value, which refers to the weight of stock plus 0.00001.

4 . Correlation analysis

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV
eturn	7.5%	19.6%	4.8%	12.0%	11.1%	1.0%	8.9%	9.1%	9.8%
tDev	14.9%	31.8%	23.2%	33.9%	39.3%	42.5%	28.1%	20.0%	31.8%
beta	1.0000	1.4228	1.0137	1.4829	1.5955	2.0067	1.0519	0.8033	1.1496
lpha	0.0000	0.0885	-0.0289	0.0081	-0.0093	-0.1410	0.0095	0.0301	0.0118
tdev	0.0%	23.8%	17.6%	25.8%	31.4%	30.3%	23.4%	16.0%	26.8%

I calculate all the required estimates for each optimization problem MM and IM based on monthly data by Solver. The result is shown in the chart above; it shows the data on these ten different types of venture capital, such as annual average returns, excess returns or risk, etc. The HA has the highest Annual Average return and risk from the table.

4.1 Comparisons on Minimum Variance and Maximum Sharpe Ratios

In this part, we have a closer look at the differences between the Markowitz and Index Model(under five constraints). We compared the minimum variance, which we named MinVar in our paper, and maximum Sharpe ratios, which we named MaxSharpe, between the two models. The two graphs below show MinVar and MaxSharpe for each stock for both MM and IM.

MM (Constr1):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	110.97%	-9.68%	5.14%	-9.90%	0.35%	-22.54%	14.06%	19.45%	-0.13%	-4.86%	-2.85%	6.72%	11.75%	0.572
MaxSharpe	39.54%	28.26%	-0.10%	0.29%	16.67%	-48.07%	19.80%	30.14%	-1.80%	7.70%	7.57%	17.59%	17.70%	0.994
MM (Constr2):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	100.00%	-8.53%	8.25%	-9.24%	0.66%	-23.03%	16.19%	22.93%	0.28%	-4.76%	-2.75%	6.97%	11.79%	0.591
MaxSharpe	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%	22.07%	21.33%	1.035
MM (Constr3):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	111.46%	-9.91%	5.27%	-10.07%	0.60%	-22.81%	14.06%	19.53%	-0.26%	-4.98%	-2.88%	6.69%	11.75%	0.570
MaxSharpe	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%	22.07%	21.33%	1.035
MM (Constr4):	SPX	ADBE	I 图表区	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	83.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.39%	0.00%	0.00%	0.00%	7.79%	14.61%	0.533
MaxSharpe	0.00%	50.27%	0.00%	0.00%	0.00%	0.00%	0.00%	17.56%	0.00%	19.52%	12.66%	18.24%	24.75%	0.737
MM (Constr5):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	0.00%	3.53%	34.20%	-1.99%	1.18%	-24.94%	34.79%	52.68%	5.03%	-2.86%	-1.61%	9.38%	15.45%	0.607
MaxSharpe	0.00%	50.82%	-19.51%	13.81%	36.74%	-76.96%	28.73%	49.95%	-16.54%	18.45%	14.50%	26.53%	25.98%	1.021

IM (Constr1):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	138.69%	-9.85%	0.03%	-9.68%	-8.19%	-15.39%	-0.28%	11.23%	-2.17%	-1.77%	-2.62%	6.07%	11.96%	0.508
MaxSharpe	48.18%	35.88%	-14.54%	2.04%	-0.61%	-34.85%	3.97%	28.36%	3.60%	17.73%	10.24%	18.90%	21.04%	0.898
IM (Constr2):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	100.00%	-6.58%	8.50%	-6.99%	-6.49%	-13.97%	3.74%	23.09%	0.74%	0.08%	-2.12%	6.90%	12.47%	0.553
MaxSharpe	54.94%	38.26%	-22.71%	2.98%	-2.31%	-37.58%	4.25%	28.63%	4.00%	18.66%	10.89%	19.81%	21.99%	0.901
IM (Constr3):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	144.39%	-10.70%	-0.63%	-10.37%	-8.62%	-15.69%	-1.35%	10.96%	-2.97%	-2.27%	-2.75%	5.85%	11.95%	0.490
MaxSharpe	54.94%	38.26%	-22.71%	2.98%	-2.31%	-37.58%	4.25%	28.63%	4.00%	18.66%	10.89%	19.81%	21.99%	0.901
IM (Constr4):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	83.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.39%	0.00%	0.00%	0.00%	7.79%	14.61%	0.533
MaxSharpe	0.00%	40.84%	0.00%	0.00%	0.00%	0.00%	0.00%	26.27%	0.00%	20.52%	12.38%	17.28%	23.35%	0.740
IM (Constr5):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	0.00%	2.68%	29.08%	0.61%	-1.69%	-10.08%	15.22%	50.40%	9.09%	5.38%	-0.70%	9.26%	16.64%	0.556
MaxSharpe	0.00%	52.54%	-20.56%	8.64%	0.79%	-41.18%	9.50%	42.11%	8.47%	25.23%	14.46%	23.79%	26.68%	0.892

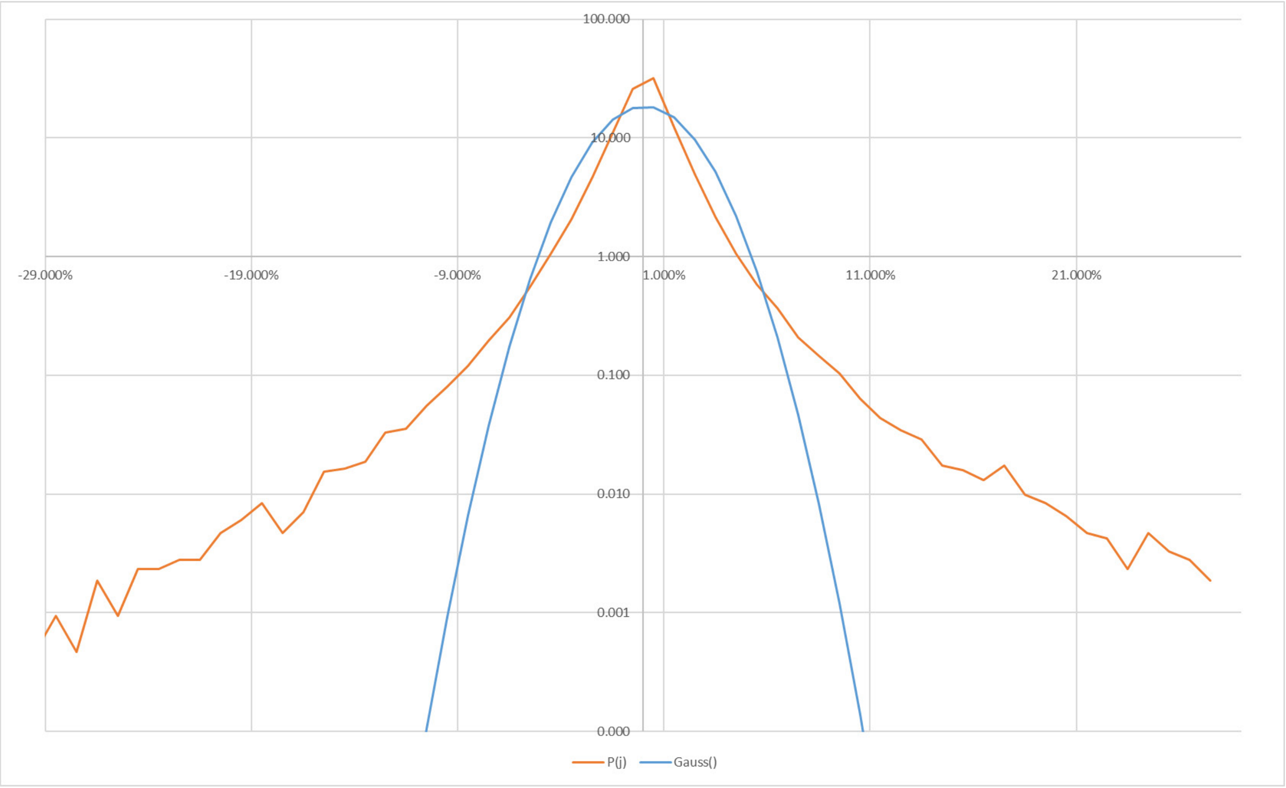
4.2 Comparison of Stocks with Normal Distribution

To compare the proximity of daily and monthly returns to the normal distribution, I calculate the probability density value and expected probability density function value and plot them out using the following formula:

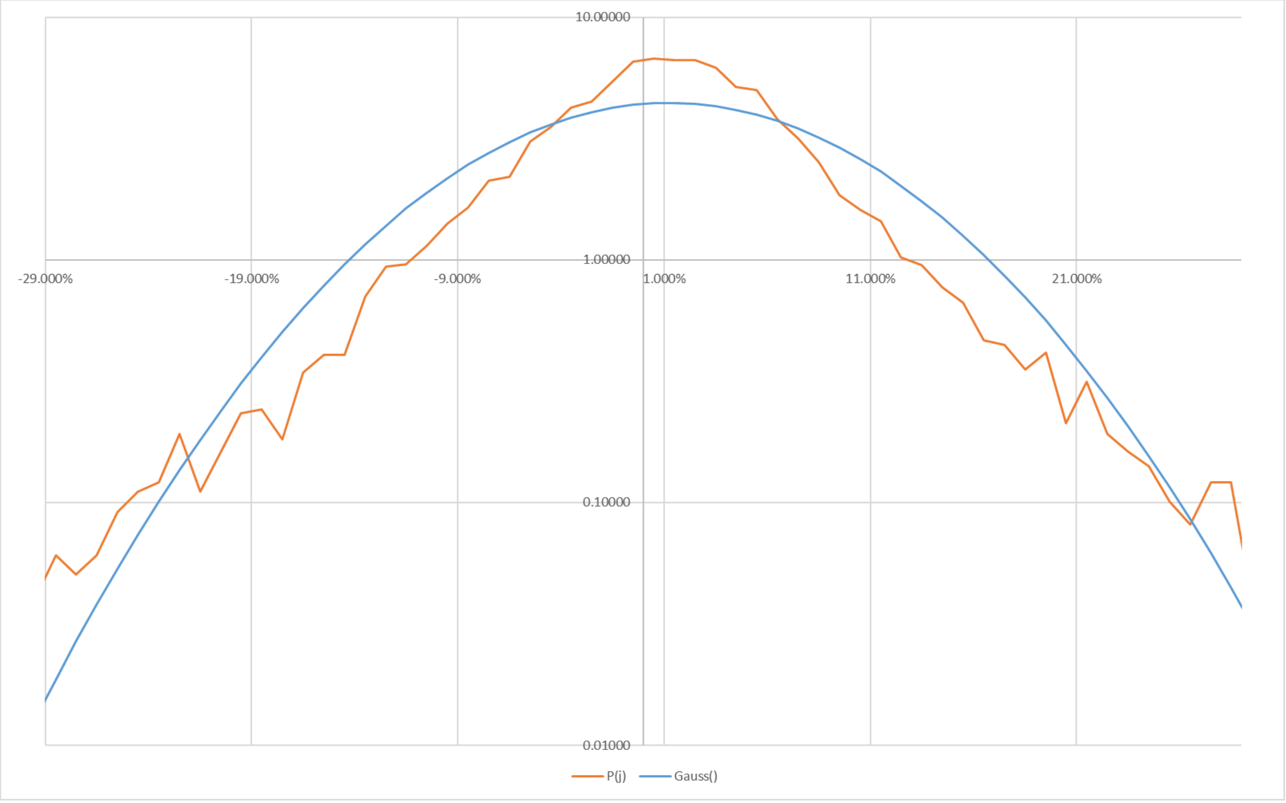
$$P_j = \frac{1}{\Delta r}\frac{N_j}{N}$$

I have depicted the corresponding curve for comparison between daily and monthly.

Comparison to Normal: Daily



Comparison to Normal: Monthly



Overall, the comparison results from the visualized data show that monthly data is more concentrated than daily data. Despite the fluctuations of monthly data, it revolves around the standard probability density curve. Therefore, I will use monthly data as the input for the following models and calculations as they are more normal.

5 . Data analytics

5.1 Comparison between IM and MM

We obtain the overall data map after the daily data processing and analysis summary. We organized the disordered daily data into a monthly quantitative table in Excel to normalize it. Here are some images of the data.

IM Efficient Frontier:

StdDev	Constr1	Constr2	Constr3	Constr4	Constr5
10.0%					
10.5%					
11.0%					
11.5%					
12.0%			6.7%		
12.5%	8.6%	7.5%	8.6%		
13.0%	9.7%	9.5%	9.7%		
13.5%	10.6%	10.5%	10.6%		
14.0%	11.4%	11.3%	11.4%		
14.5%	12.1%	12.1%	12.1%		
15.0%	12.7%	12.7%	12.7%	8.7%	
15.5%	13.3%	13.3%	13.3%	9.5%	
16.0%	13.9%	13.9%	13.9%	10.2%	
16.5%	14.5%	14.5%	14.5%	10.8%	
17.0%	15.0%	15.0%	15.0%	11.4%	6.8%
17.5%	15.5%	15.5%	15.5%	12.0%	13.0%
18.0%	16.0%	16.0%	16.0%	12.5%	14.0%
18.5%	16.5%	16.5%	16.5%	13.0%	14.9%
19.0%	17.0%	17.0%	17.0%	13.5%	15.6%
19.5%	17.5%	17.5%	17.5%	14.0%	16.3%
20.0%	18.0%	18.0%	18.0%	14.5%	17.0%
20.5%	18.4%	18.5%	18.5%	14.9%	17.6%
21.0%	18.9%	18.9%	18.9%	15.4%	18.2%
21.5%	19.3%	19.4%	19.4%	15.8%	18.7%
22.0%	19.8%	19.8%	19.8%	16.2%	19.3%
22.5%	20.2%	20.3%	20.3%	16.6%	19.8%
23.0%	20.6%	20.7%	20.7%	17.0%	20.3%
23.5%	21.0%	21.2%	21.2%	17.4%	20.8%
24.0%	21.5%	21.6%	21.6%	17.7%	21.3%
24.5%	21.9%	22.0%	22.0%	18.1%	21.8%
25.0%	22.3%	22.5%	22.5%	18.4%	22.3%
25.5%	22.7%	22.9%	22.9%	18.8%	22.7%
26.0%	23.1%	23.3%	23.3%	19.1%	23.2%
26.5%	23.5%	23.7%	23.7%	19.4%	23.6%
27.0%	23.9%	24.2%	24.2%	19.7%	24.1%
27.5%	24.3%	24.6%	24.6%	20.0%	24.5%
28.0%	24.7%	25.0%	25.0%	20.3%	25.0%
28.5%	25.1%	25.3%	25.4%	20.6%	25.4%
29.0%	25.4%	25.8%	25.8%	20.8%	25.8%
29.5%	25.8%	26.3%	26.3%	21.0%	26.2%
30.0%	26.1%	26.7%	26.7%	21.2%	26.7%
30.5%	26.5%	27.1%	27.1%	21.3%	27.1%
31.0%	26.8%	27.5%	27.5%	21.5%	27.5%
31.5%	27.1%	27.9%	27.9%	21.6%	27.9%
32.0%	27.5%	28.3%	28.3%	21.8%	28.3%
32.5%	27.8%	28.7%	28.7%	21.9%	28.7%
33.0%	28.1%	29.1%	29.1%	22.0%	29.1%
33.5%	28.4%	29.5%	29.5%	22.2%	29.5%
34.0%	28.7%	29.9%	29.9%	22.3%	29.9%
34.5%	29.1%	30.3%	30.3%	22.4%	30.3%
35.0%	29.4%	30.7%	30.7%	22.5%	30.7%
35.5%	29.7%	31.1%	31.1%	22.6%	31.1%
36.0%	30.0%	31.5%	31.5%	22.7%	31.5%

IM Inefficient Frontier:

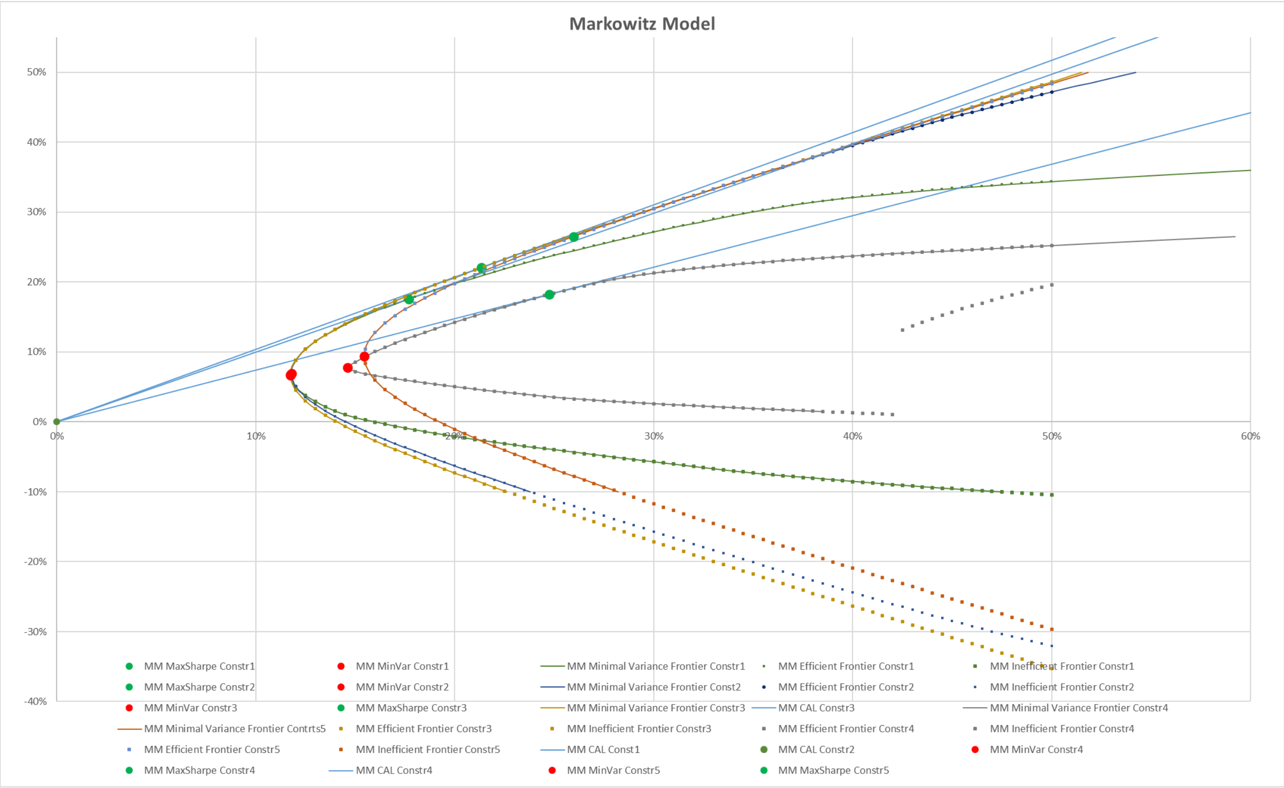
StdDev	Constr1	Constr2	Constr3	Constr4	Constr5
10.0%					
10.5%					
11.0%					
11.5%					
12.0%	5.5%		5.2%		
12.5%	3.9%	6.3%	3.1%		
13.0%	3.1%	4.3%	2.0%		
13.5%	2.4%	3.3%	1.1%		
14.0%	1.8%	2.5%	0.3%		
14.5%	1.3%	1.7%	-0.4%		
15.0%	0.9%	1.1%	-1.0%	7.3%	
15.5%	0.6%	0.5%	-1.6%	6.9%	
16.0%	0.2%	-0.1%	-2.2%	6.6%	
16.5%	-0.1%	-0.6%	-2.8%	6.4%	
17.0%	-0.3%	-1.2%	-3.3%	6.2%	6.8%
17.5%	-0.6%	-1.7%	-3.8%	6.0%	5.5%
18.0%	-0.8%	-2.2%	-4.3%	5.8%	4.5%
18.5%	-1.1%	-2.6%	-4.8%	5.6%	3.6%
19.0%	-1.3%	-3.1%	-5.3%	5.5%	2.9%
19.5%	-1.5%	-3.6%	-5.8%	5.3%	2.2%
20.0%	-1.7%	-4.0%	-6.3%	5.1%	1.5%
20.5%	-1.9%	-4.4%	-6.8%	5.0%	0.9%
21.0%	-2.1%	-4.9%	-7.2%	4.8%	0.3%
21.5%	-2.3%	-5.3%	-7.7%	4.6%	-0.2%
22.0%	-2.5%	-5.7%	-8.1%	4.5%	-0.8%
22.5%	-2.7%	-6.2%	-8.6%	4.3%	-1.3%
23.0%	-2.9%	-6.6%	-9.0%	4.2%	-1.8%
23.5%	-3.1%	-7.0%	-9.5%	4.0%	-2.3%
24.0%	-3.2%	-7.4%	-9.9%	3.9%	-2.8%
24.5%	-3.4%	-7.8%	-10.3%	3.7%	-3.3%
25.0%	-3.6%	-8.2%	-10.8%	3.6%	-3.7%
25.5%	-3.8%	-8.6%	-11.2%	3.5%	-4.2%
26.0%	-3.9%	-9.0%	-11.6%	3.3%	-4.7%
26.5%	-4.1%	-9.4%	-12.0%	3.2%	-5.1%
27.0%	-4.3%	-9.8%	-12.5%	3.1%	-5.6%
27.5%	-4.4%	-10.2%	-12.9%	3.1%	-6.0%
28.0%	-4.6%	-10.6%	-13.3%	3.0%	-6.4%
28.5%	-4.8%	-11.0%	-13.7%	2.9%	-6.9%
29.0%	-4.9%	-11.4%	-14.1%	2.8%	-7.3%
29.5%	-5.1%	-11.7%	-14.6%	2.7%	-7.7%
30.0%	-5.2%	-12.1%	-15.0%	2.6%	-8.1%
30.5%	-5.4%	-12.5%	-15.4%	2.6%	-8.6%
31.0%	-5.6%	-12.9%	-15.8%	2.5%	-9.0%
31.5%	-5.7%	-13.3%	-16.2%	2.4%	-9.4%
32.0%	-5.9%	-13.6%	-16.6%	2.4%	-9.8%
32.5%	-6.0%	-14.0%	-17.0%	2.3%	-10.2%
33.0%	-6.2%	-14.4%	-17.4%	2.2%	-10.6%
33.5%	-6.3%	-14.8%	-17.8%	2.1%	-11.0%
34.0%	-6.5%	-15.1%	-18.2%	2.1%	-11.4%
34.5%	-6.6%	-15.5%	-18.6%	2.0%	-11.8%
35.0%	-6.8%	-15.9%	-19.0%	1.9%	-12.2%
35.5%	-6.9%	-16.3%	-19.4%	1.9%	-12.6%
36.0%	-7.1%	-16.6%	-19.8%	1.8%	-13.0%

MM Efficient Frontier:					
StdDev	Constr1	Constr2	Constr3	Constr4	Constr5
10.0%					
10.5%					
11.0%					
11.5%					
12.0%	8.8%	8.8%	8.8%		
12.5%	10.4%	10.4%	10.4%		
13.0%	11.5%	11.5%	11.5%		
13.5%	12.4%	12.4%	[illegible]		
14.0%	13.2%	13.3%	13.3%		
14.5%	13.9%	14.0%	14.0%		
15.0%	14.6%	14.7%	14.7%	8.6%	
15.5%	15.2%	15.4%	15.4%	9.4%	10.4%
16.0%	15.8%	16.1%	16.1%	10.0%	12.8%
16.5%	16.4%	16.7%	16.7%	10.7%	14.1%
17.0%	16.9%	17.3%	17.3%	11.2%	15.2%
17.5%	17.4%	17.9%	17.9%	11.8%	16.1%
18.0%	17.9%	18.5%	18.5%	12.3%	17.0%
18.5%	18.4%	19.0%	19.0%	12.8%	17.7%
19.0%	18.8%	19.6%	19.6%	13.3%	18.5%
19.5%	19.3%	20.1%	20.1%	13.8%	19.1%
20.0%	19.8%	20.7%	20.7%	14.3%	19.8%
20.5%	20.2%	21.2%	21.2%	14.7%	20.4%
21.0%	20.6%	21.7%	21.7%	15.2%	21.1%
21.5%	21.1%	22.2%	22.2%	15.6%	21.7%
22.0%	21.5%	22.8%	22.8%	16.0%	22.2%
22.5%	21.9%	23.3%	23.3%	16.4%	22.8%
23.0%	22.3%	23.8%	23.8%	16.9%	23.4%
23.5%	22.7%	24.3%	24.3%	17.3%	23.9%
24.0%	23.1%	24.8%	24.8%	17.7%	24.5%
24.5%	23.5%	25.3%	25.3%	18.0%	25.0%
25.0%	23.8%	25.7%	25.7%	18.4%	25.5%
25.5%	24.2%	26.2%	26.2%	18.8%	26.0%
26.0%	24.5%	26.7%	26.7%	19.1%	26.5%
26.5%	24.9%	27.2%	27.2%	19.5%	27.0%
27.0%	25.2%	27.7%	27.7%	19.8%	27.6%
27.5%	25.5%	28.2%	28.2%	20.1%	28.1%
28.0%	25.9%	28.6%	28.6%	20.4%	28.6%
28.5%	26.2%	29.1%	29.1%	20.7%	29.0%
29.0%	26.5%	29.6%	29.6%	20.9%	29.5%
29.5%	26.8%	30.1%	30.1%	21.1%	30.0%
30.0%	27.2%	30.5%	30.5%	21.3%	30.5%
30.5%	27.5%	31.0%	31.0%	21.5%	31.0%
31.0%	27.8%	31.5%	31.5%	21.7%	31.4%
31.5%	28.1%	31.9%	31.9%	21.8%	31.9%
32.0%	28.4%	32.4%	32.4%	22.0%	32.4%
32.5%	28.7%	32.9%	32.9%	22.1%	32.8%
33.0%	29.0%	33.3%	33.3%	22.3%	33.3%
33.5%	29.3%	33.8%	33.8%	22.4%	33.8%
34.0%	29.5%	34.2%	34.2%	22.5%	34.2%
34.5%	29.8%	34.7%	34.7%	22.6%	34.7%
35.0%	30.1%	35.2%	35.2%	22.7%	35.2%
35.5%	30.3%	35.6%	35.6%	22.9%	35.6%
36.0%	30.5%	36.0%	36.1%	23.0%	36.1%

MM Inefficient Frontier:					
StdDev	Constr1	Constr2	Constr3	Constr4	Constr5
10.0%					
10.5%					
11.0%					
11.5%					
12.0%	5.0%	5.1%	4.6%		
12.5%	3.8%	3.6%	3.0%		
13.0%	2.9%	2.5%	1.9%		
13.5%	2.2%	1.6%	0.9%		
14.0%	1.6%	0.8%	0.1%		
14.5%	1.1%	0.0%	-0.6%		
15.0%	0.6%	-0.6%	-1.4%	7.2%	
15.5%	0.3%	-1.3%	-2.0%	6.9%	8.4%
16.0%	0.0%	-1.9%	-2.7%	6.6%	6.0%
16.5%	-0.3%	-2.5%	-3.3%	6.4%	4.6%
17.0%	-0.6%	-3.1%	-3.9%	6.2%	3.6%
17.5%	-0.9%	-3.6%	-4.5%	6.0%	2.6%
18.0%	-1.1%	-4.2%	-5.1%	5.8%	1.8%
18.5%	-1.4%	-4.7%	-5.7%	5.6%	1.0%
19.0%	-1.6%	-5.3%	-6.2%	5.4%	0.3%
19.5%	-1.9%	-5.8%	-6.7%	5.2%	-0.4%
20.0%	-2.1%	-6.3%	-7.3%	5.1%	-1.0%
20.5%	-2.3%	-6.8%	-7.8%	4.9%	-1.7%
21.0%	-2.5%	-7.3%	-8.3%	4.7%	-2.3%
21.5%	-2.7%	-7.8%	-8.9%	4.6%	-2.9%
22.0%	-2.9%	-8.3%	-9.4%	4.4%	-3.5%
22.5%	-3.1%	-8.8%	-9.9%	4.2%	-4.0%
23.0%	-3.3%	-9.2%	-10.4%	4.1%	-4.6%
23.5%	-3.4%	-9.7%	-10.9%	3.9%	-5.2%
24.0%	-3.6%	-10.2%	-11.4%	3.8%	-5.7%
24.5%	-3.8%	-10.7%	-11.9%	3.6%	-6.2%
25.0%	-4.0%	-11.1%	-12.4%	3.5%	-6.8%
25.5%	-4.2%	-11.6%	-12.9%	3.4%	-7.3%
26.0%	-4.3%	-12.1%	-13.3%	3.3%	-7.8%
26.5%	-4.5%	-12.5%	-13.8%	3.2%	-8.3%
27.0%	-4.7%	-13.0%	-14.3%	3.1%	-8.8%
27.5%	-4.9%	-13.4%	-14.8%	3.0%	-9.3%
28.0%	-5.0%	-13.9%	-15.3%	2.9%	-9.8%
28.5%	-5.2%	-14.3%	-15.7%	2.8%	-10.3%
29.0%	-5.4%	-14.8%	-16.2%	2.8%	-10.8%
29.5%	-5.5%	-15.2%	-16.7%	2.7%	-11.2%
30.0%	-5.7%	-15.7%	-17.1%	2.6%	-11.7%
30.5%	-5.9%	-16.1%	-17.6%	2.5%	-12.2%
31.0%	-6.0%	-16.6%	-18.1%	2.5%	-12.7%
31.5%	-6.2%	-17.0%	-18.5%	2.4%	-13.2%
32.0%	-6.4%	-17.5%	-19.0%	2.3%	-13.6%
32.5%	-6.5%	-17.9%	-19.5%	2.2%	-14.1%
33.0%	-6.7%	-18.3%	-19.9%	2.2%	-14.6%
33.5%	-6.8%	-18.8%	-20.4%	2.1%	-15.0%
34.0%	-7.0%	-19.2%	-20.9%	2.1%	-15.5%
34.5%	-7.1%	-19.6%	-21.3%	2.0%	-15.9%
35.0%	-7.3%	-20.1%	-21.8%	1.9%	-16.4%
35.5%	-7.4%	-20.5%	-22.2%	1.9%	-16.9%
36.0%	-7.6%	-21.0%	-22.7%	1.8%	-17.3%

By utilizing the theory and tools discussed above, we finally export two graphs: one represents the Markowitz Model(MM), and another one represents the Index Model(IM). Standard deviation is on the x-axis, while returns are on the y-axis. The two graphs below show the changes in standard deviation and returns under different constraints and how returns connect with standard deviation under these constraints.

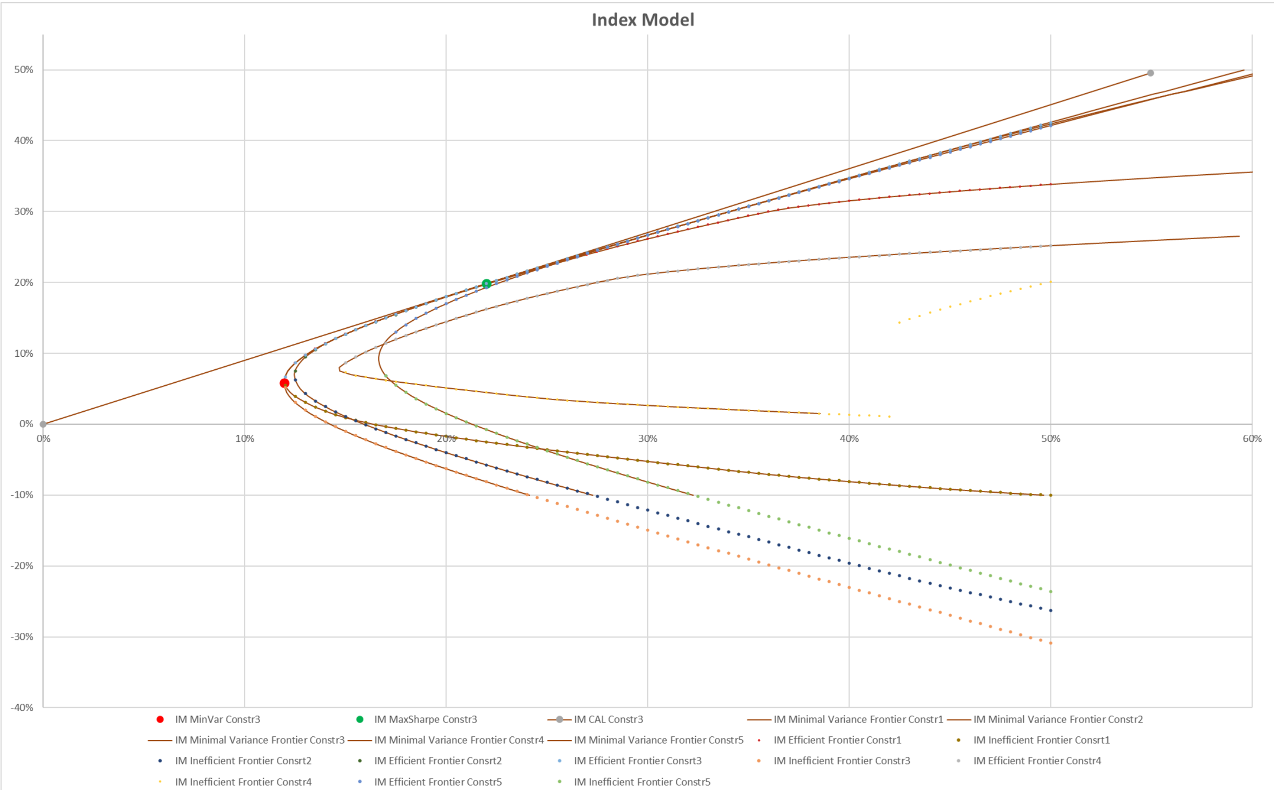
(1) Markowitz Model(MM)



Analyzing the Markowitz Model, we can summarize that one stock may have two different returns under the same standard deviation. Under constraint 1 $\sum_{i=1}^{11}|w_i|\leqslant 2$, higher returns for one specific standard deviation are the efficient frontier, while lower returns are the inefficient frontier. The minimal variance frontier is a curve connecting all points representing the efficient and inefficient frontier.

When considering constraint 2, we notice that the frontier curve consisting of efficient and inefficient frontier points has a steeper slope. That means under constraint 2, the ten stocks have higher returns when the standard deviation is the same. Constraint 3 makes the frontier curve have a steeper slope than the curve under constraint 2. The frontier curve under constraint 4 has the flattest slope among these curves.

(2)Index Model(IM)



In the graph on IM, we notice that frontier curves have a similar shape to the ones in the graph of MM. Frontier curves under constraint 4 have the flattest slope. Frontier curves have steeper slopes from constraint 1 to 3. However, the Index Model makes covariance estimation easier and improves our analysis efficiency.

6 . Conclusion

In this paper, I compared the Markowitz Model to the Index Model by selecting ten stocks from three different sectors to demonstrate how these two models can show the risk and return of a portfolio's asset allocation. To examine the normality of the data, I calculated the basic statistics and showed the probability density function of the daily and monthly data of the ten stocks. Monthly data presented a higher proximity to a normal distribution than daily data. By comparing and analyzing the results from the Markowitz Model (MM) and Index Model (IM) under varied constraints, it emerges that despite certain assumptions of the models not being fully satisfied, the IM model proves to be a reasonably good approximation of the MM model. Notably, the MM model tends to slightly supersede the IM model in performance. Compared to the Markowitz Model, the Index Model has an easier calculation process. However, the problem is that the model can't reflect the real risk in the market. The index Model simplifies risks into systematic risks and firm-specific risks. If we hope to reflect the ten stocks' real risk, the Markowitz Model is definitely a better choice.

Bibliography:

1) Index model
Single Index Model - What It Is, Formula, Assumptions, Portfolio (wallstreetmojo.com)
Single-index model - Wikipedia
2)The Full Markowitz Model
(zhihu.com)
Markowitz Model of Risk-Return Optimization | Assumptions (fundamentalsofaccounting.org)
3)Yahoo Finance - Stock Market Live, Quotes, Business & Finance News
4)Stocks information
Adobe Inc. - Wikipedia
Wikipedia, the free encyclopedia