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Bridging the Disparity: An Optimization Model for Equitable Allocation of Urban and Rural Educational Resources in Western China

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Abstract:

The persistent disparity in educational resources between urban and rural areas in western China remains a critical barrier to achieving educational equity. This study addresses the critical issue by developing an innovative optimization model. The model aims to allocate financial, human, and physical resources effectively to minimize the educational quality gap between these regions. Using information from the China Education Panel Survey (CEPS) and concentrating on Xinjiang as a representative instance, our design integrates budget constraints, minimum resource requirements, and a unique regional prioritization factor. The goal of the objective function is to reduce the squared disparity in educational quality between city and rural regions. It thinks about variables such as Teacher-Student Ratio (TSR), Per Capita Investment (PCI), and Teaching Equipment Perfection (TEP). Our searchings for show that strategic resource distribution can considerably lessen educational inequalities in between metropolitan and country setups, especially by focusing on underdeveloped rural regions. Sensitivity analysis shows that customizing the local prioritization specification makes it possible for versatile policy strategies, ranging from equitable distribution to plainly sustaining rural regions. This study presents a pragmatic structure for boosting resource allotment in education. It supplies vital understandings for policymakers aiming to promote an extra equitable educational environment in Western China.

Keywords: Education;Urban;Rural;Western.

1. Introduction

The space in educational resources between urban and rural regions in western China remains a consistent obstacle. This difference dramatically affects both the high quality and equity of education and learning. The disproportionate allotment of sources, including skilled educators, adequate infrastructure, and financial support, escalates the instructional divide. This situation perpetuates social and economic disparities between urban and country communities. Remedying this discrepancy is vital for acquiring instructional equity. It likewise promotes social cohesion and sustains sustainable development in the region.

Urban regions in western China usually take pleasure in boosted facilities, access to qualified educators, and considerable financial backing. These benefits cause improved academic achievements and improved educational results for students in city areas. However, country educational institutions frequently face resource restrictions, substandard facilities, and an absence of certified instructors. These challenges add to lessened academic performance and restricted opportunities for trainees in backwoods. This educational gap dramatically adds to the ongoing cycle of poverty. It restricts upward social mobility for rural neighborhoods.

Previous studies have highlighted the immediate requirement for a more fairer allowance of educational resources. For example, a research study [1] showed that focused resource distribution could substantially enhance educational results in country establishments. On the other hand, a research [2] highlighted the need of optimizing source allotment to achieve better equity in education and learning. Regardless of these searchings, the predicament of how to ideally distribute resources under limited conditions remains to be a dynamic field of investigation.

This paper attends to the problem by formulating an extensive optimization framework targeted at the distribution of educational resources in both the city and backwoods of western China. Our version emphasizes 3 important groups of possessions: fiscal resources, human capital (teachers), and material resources (infrastructure and supplies). The goal of our design's objective function is to lessen the squared disparity in educational quality between city and rural regions, thinking about variables such as Teacher-Student Ratio (TSR), Per Capita Investment (PCI), and the competence of Teaching Equipment (TEP). One unique attribute of our design is the assimilation of a regional prioritization factor, making it possible for flexible policy strategies in resource distribution. This aspect equips policymakers to check out varied scenarios, ranging from achieving equilibrium between metropolitan and rural regions to putting a substantial focus on country advancement.

The research study employs information sourced from the China Education Panel Survey (CEPS), concentrating on Xinjiang as an essential case study for the western region of China. By integrating empirical data and imposed restrictions, such as budgetary restrictions and essential resource allocations, our version delivers a carefully validated approach to resource distribution that can directly assist policy-making processes.

The importance of this research is rooted in its ability to assist policymakers in creating evidence-based decisions focused on minimizing the educational inequalities existing between city and rural regions.This study provides a quantitative framework for resource distribution, thus advancing the overarching objective of attaining educational equity and enhancing the overall quality of education in Western China. The results of this research may act as a significant reference for upcoming policy formulation, potentially impacting other areas encountering comparable difficulties.

In conclusion, our optimization model for the distribution of educational resources is a vital advancement in tackling the educational inequities that exist between urban and rural regions in western China. This research will investigate the evolution, utilization, and repercussions of this framework, intending to establish a solid basis for a more equitable and efficient allocation of resources within the educational system of the region.

2. Related Works

The issue of educational resource allocation has been the subject of extensive research within China and internationally. Researchers have analyzed multiple dimensions of this topic, encompassing the inequalities between metropolitan and rural regions, the efficacy of diverse resource distribution methodologies, and the formulation of optimization frameworks to inform policy-making. This segment examines the principal contributions found in the literature that are most pertinent to this research.

2.1 Urban-Rural Educational Disparities

Investigations into the educational inequities between urban and rural locales have persistently demonstrated that rural regions, especially in developing countries, experience a pronounced deficiency in resources when compared to their urban counterparts. In the Chinese context, [7,8] explores the underlying causes and consequences of these disparities, emphasizing that the disproportionate allocation of qualified educators and educational resources is a significant contributor to the performance gap observed between urban and rural students. Their research emphasizes the necessity for more strategic resource distribution approaches to tackle these imbalances and enhance academic achievements in rural regions.

Similarly, [9, 10] performed an in-depth examination of the distribution of educational resources in western China, emphasizing the efficacy of governmental strategies designed to mitigate the disparities between urban and rural areas. While recent policies have resulted in certain advancements, considerable inequalities remain, especially in isolated and underserved areas. [6, 3, 4, 5] The study highlighted the necessity of creating more sophisticated allocation frameworks that address the distinct challenges encountered by rural educational institutions.

2.2 Optimization Models in Resource Allocation

Optimization models have been extensively employed across diverse disciplines to tackle issues related to resource allocation. In the field of education, these models have been utilized to enhance the allocation of scarce resources in order to attain targeted objectives, such as optimizing student achievement or reducing inequalities in educational standards. A significant research endeavor outlined in [11] formulated a linear programming framework aimed at enhancing the distribution of educational resources throughout various regions in Spain. Their model aimed to optimize educational results by incorporating variables such as institutional size, student-to-teacher ratios, and available financial resources. The findings of their research offered significant perspectives on the application of optimization models to enhance resource distribution within the educational sector.

In the context of China's educational landscape, [7, 8, 9] established a multi-objective optimization model aimed at effectively distributing educational resources in rural regions. Their model sought to harmonize various goals, such as equity, efficiency, and sustainability, through an analysis of the distinct requirements of rural areas. The research illustrated that optimization models can significantly influence policy-making, especially in areas with constrained resources.

Despite advancements in optimization models for educational resource allocation, a gap persists in addressing urban-rural disparities in western China. Existing models often overlook rural needs, concentrating instead on national strategies or urban contexts. This research aims to address the urban-rural educational gap in western China by creating a context-specific optimization model for improved resource allocation.

3. Model Development

In this section, we present a comprehensive optimization model designed to address the critical challenge of allocating educational resources between urban and rural areas in western China. This model represents a significant step towards reducing the educational quality gap that has long persisted between these regions, a disparity that has far-reaching implications for social equity and economic development.

Our approach recognizes that educational resource allocation is a complex issue necessitating careful consideration of multiple factors. This model equips policymakers and educators with a tool for informed, data-driven decisions to achieve equitable educational outcomes.

3.1 Assumptions.

The core of a solid model is its assumptions. In our case, these assumptions are carefully crafted to balance theoretical rigor with practical applicability. Each assumption is grounded in empirical evidence and reflects the realities of the educational landscape in Western China.

1. Resource Types: We categorize educational resources into three main types: financial resources, human resources (qualified teachers), and physical resources (infrastructure and materials). This tripartite classification is not arbitrary; rather, it is based on extensive research ([12,13,14]) that identifies these as the most critical inputs for educational quality. By focusing on these three categories, we ensure that our model captures the essential elements that drive educational outcomes.

2. Budget Constraints: The total budget available for allocation is fixed and cannot be exceeded. This assumption reflects the real-world constraints faced by policymakers and educational administrators. In most cases, especially in government-funded programs, educational budgets are predetermined and inflexible. By incorporating this constraint, our model provides solutions that are not only optimal but also feasible within existing financial limitations.

3. Minimum Resource Requirements: Each school, whether urban or rural, has a minimum requirement for each category of resources to maintain basic educational standards. This assumption is crucial for ensuring that our model doesn't propose solutions that might inadvertently compromise the quality of education in any region. It acknowledges that there's a baseline level of resources needed for effective learning, such as a minimum number of teachers per student or basic infrastructure needs.

4. Regional Prioritization: Certain rural areas are prioritized based on their level of educational underdevelopment, requiring higher per-student resources than others. This assumption reflects the reality that not all rural areas face the same level of educational challenges. Some regions, due to historical, geographical, or socioeconomic factors, may be more disadvantaged than others. By incorporating this prioritization, our model allows for a more nuanced approach to resource allocation, focusing additional resources where they are most needed to achieve educational parity.

These assumptions, while necessary for model simplification, are carefully chosen to ensure that our model remains both theoretically sound and practically relevant. They allow us to create a framework that can effectively guide resource allocation decisions while acknowledging the complexities of the real-world educational landscape in Western China.

3.2 Variables and Parameters

To translate our conceptual model into a mathematical framework, we need to define a set of variables and parameters. These elements form the building blocks of our optimization model, allowing us to quantify and analyze the complex relationships between resource allocation and educational quality.

3.2.1 Decision Variables

The core of our model revolves around the decision variables, which represent the choices available to policymakers in allocating resources:

· x_{ij} : The amount of resource type j allocated to a region i, where i can be urban (U) or rural (R), i.e., $i \in \{U, R\}$, and j represents different types of resources (e.g., financial, human, physical), i.e., $j \in \{$ fin, hum, phys $\}$.

These decision variables are the levers that our model will adjust to find the optimal allocation strategy. They directly influence the educational quality in each region and are the primary focus of the optimization process.

3.2.2 Intermediate Variables and Parameters

To bridge the gap between raw resource allocation and educational quality, we introduce several intermediate variables and parameters. These elements allow us to model the complex relationships between inputs (resources) and outputs (educational quality) more accurately:

· Educational Quality Indicators: Q_i : This variable represents the overall educational quality in a region i (urban or rural). It is a composite measure calculated as a function of various resource inputs, reflecting the multifaceted nature of educational quality.

- Teacher-Student Ratio (TSR):

$$TSR_i = \frac{x_{i,hum}}{S_i}$$

This ratio is a critical indicator of educational quality, reflecting the level of individual attention and support students can receive. Here, $x_{i,hum}$ represents the number of teachers allocated to region i, and S_i is the number of students in that region.

· Per Capita Investment (PCI):

$$PCI_i = \frac{x_{i,fin}}{S_i}$$

This measure captures the financial resources available per student, where $x_{i,\text{in}}$ is the total financial investment allocated to the region i. PCI is crucial for understanding the level of monetary support each student receives, which can significantly impact educational outcomes.

- Teaching Equipment Perfection (TEP):

$$TEP_i = \frac{x_{i,phys}}{E_i}$$

This ratio assesses the adequacy of physical resources and infrastructure. Here, $x_{i,\text{phys}}$ represents the physical resources allocated to region i, and E_i represents the baseline equipment needs. This measure is crucial for understanding the quality of the learning environment.

· Weights: w_j : These weights are assigned to each educational quality factor, reflecting their relative importance. They are determined based on empirical evidence and policy priorities, allowing the model to reflect the nuanced impact of different types of resources on overall educational quality.

- Parameters:

· T_j : The total available resources for allocation across all regions and resource types j. This parameter sets the overall budget constraint.

· M_{ij} : The minimum required amount of resource type j for region i. This guarantees uniform educational standards across all regions.

· S_i : The number of students in region i. This is crucial for calculating per-student resource allocation.

· E_i : Baseline equipment requirements for the region i. assist in evaluating resource adequacy.

By integrating these variables, our model captures the intricate relationship between resource allocation and educational quality, enabling a nuanced representation of the educational system in Western China.

3.3 Objective Function

The essence of our optimization model is the objective function, which aims to minimize the educational quality disparity between urban and rural areas, thus fostering a more equitable educational environment in western China. The educational quality indicator Q_i for each region i is a weighted sum of identified key factors:

$$Q_i = w_1 \cdot \mathbf{TSR}_i + w_2 \cdot \mathbf{PCI}_i + w_3 \cdot \mathbf{TEP}_i$$

This formulation enables simultaneous consideration of various educational quality aspects, with the weights (w_1, w_2, w_3) indicating each factor's relative importance. These weights can be adjusted based on policy priorities or empirical evidence, providing flexibility to the model.

Our objective function then becomes:

$$\text{Minimize} Z = (Q_U - Q_R)^2$$

Where:

$$Z = \left(w_1 \cdot \left(\frac{x_{U,hum}}{S_U} - \frac{x_{R,hum}}{S_R} \right) + w_2 \cdot \left(\frac{x_{U,fin}}{S_U} - \frac{x_{R,fin}}{S_R} \right) + w_3 \cdot \left(\frac{x_{U,phys}}{E_U} - \frac{x_{R,phys}}{E_R} \right) \right)^2$$

By minimizing the squared difference between urban and rural educational quality, we ensure that our model focuses on reducing disparities. The use of a squared term emphasizes larger gaps, encouraging the model to address significant inequalities more aggressively. This objective function encapsulates our goal of creating a more balanced educational system. It drives the optimization process towards solutions that allocate resources in a way that narrows the urban-rural education gap, promoting greater equity and social justice.

3.4 Constraints

While our objective function defines what we aim to achieve, the constraints of our model ensure that the solutions we generate are realistic and implementable. These constraints reflect the practical limitations and requirements of the resource allocation problem in Western China's educational context.

· Total Resource Constraint: This constraint ensures that we don't allocate more resources than are available:

$$\sum_i x_{ij} = T_j \forall j$$

This reflects the reality of limited budgets and resources in educational planning. It forces the model to make trade-offs and prioritize allocations effectively, mirroring the challenges faced by real-world policymakers.

· Basic Demand Constraint: This constraint guarantees that each region receives at least the minimum required amount of each resource type:

$x_{ij} \geqslant M_{ij} \forall i, j$

By enforcing this constraint, we ensure that our model doesn't propose solutions that might compromise basic educational standards in any region. Maintaining a consistent standard of educational quality in both urban and rural areas is essential.

· Policy Limitations: We enforce additional restrictions per policy requirements, such as focusing on rural development progress:

$\sum_j ?x_{Rj} \geqslant \lambda \cdot \sum_j ?x_{Uj}$

This constraint reflects the goals of policy, ensuring equitable resource allocation between rural and urban areas. The parameter can be modified to indicate varying degrees of rural prioritization, enabling policymakers to investigate different scenarios.

· Non-negativity Limitation: This constraint guarantees nonnegative resource allocations:

$x_{ij} \geqslant 0 \forall i, j$

This constraint, while apparent, is essential for ensuring our model yields practical solutions. Negative resource allocations are impractical, and this guideline maintains our model's realism.

These constraints establish a framework for our optimization, ensuring solutions are both mathematically optimal and practically feasible, aligned with policy objectives.

3.5 Comprehensive Model

Integrating the objective function with the constraints, we now present our complete optimization model:

$$MinimizeZ = \left(w_1 \cdot \left(\frac{x_{U,hum}}{S_U} - \frac{x_{R,hum}}{S_R} \right) + w_2 \cdot \left(\frac{x_{U,fin}}{S_U} - \frac{x_{R,fin}}{S_R} \right) + w_3 \cdot \left(\frac{x_{U,phys}}{E_U} - \frac{x_{R,phys}}{E_R} \right) \right)^2$$

$$\text{subjectto} \begin{cases} \sum_i x_{ij} = T_j \forall j \\ x_{ij} \geqslant M_{ij} \forall i, j \\ \sum_j x_{Rj} \geqslant \lambda \cdot \sum_j x_{Uj} \\ x_{ij} \geqslant 0 \forall i, j \end{cases} \quad (1)$$

This model effectively addresses the urban-rural educational gap in western China by minimizing disparities in educational quality while considering practical constraints and policy priorities, thereby optimizing resource allocation.

This model's strength is its flexible comprehensiveness, accommodating diverse resources, regional variations, and policy-driven modifications. Simultaneously, it stays rooted in the practicalities of educational administration, guaranteeing that solutions are theoretically sound and feasible. Through this model, policymakers and educators can effectively allocate limited resources, leveraging data-driven insights for equitable educational outcomes and fostering the development of western China.

The following section will address model implementation, present results from various scenarios, and examine the implications for educational policy in Western China.

4 Data Collection and Model Solving Method

This section outlines the estimation of key parameters for the optimization model using available data, grounded in relevant literature and reliable sources, with justifications for the selected values.

4.1 Parameter Estimation

This section focuses on parameter estimation for our model, as their accuracy and reliability are vital to effectively tackling the urban-rural educational gap in western China. To guarantee the reliability of our estimates, we have utilized a comprehensive array of data from credible sources and pertinent literature.

Our main data source is the China Education Panel Survey (CEPS) [24], a comprehensive longitudinal study conducted by the National Survey Research Center (NSRC) at Renmin University of China. The CEPS is fundamental to educational research in China, offering a comprehensive, nationally representative dataset from 112 schools, 438 classes, and around 20,000 stakeholders, including students, parents, teachers, and administrators.

The CEPS highlights the complex interconnections between families, schools, communities, and social structures in influencing educational outcomes, offering essential insights for academic research and policy development in China's education system.

For our specific focus on western China, we have concentrated on data from Xinjiang as a representative case study. Xinjiang, with its unique demographic and geographic characteristics, serves as an excellent microcosm of the broader educational challenges faced in Western China. We have supplemented the CEPS data with statistics from the Xinjiang government (Xinjiang Bureau of Statistics, 2023) [22, 23] to ensure a comprehensive and up-to-date picture.

Based on these sources, we have derived the following parameter estimates:

- Student Population:

· $S_U = 32.32$ million (Number of students in urban regions)

· $S_R = 24.38$ million (Number of students in rural regions)

- Teaching Equipment Baseline:

· $E_U = 1$ (Baseline equipment needed in urban regions)

· $E_R = 0.8$ (Baseline equipment needed in rural regions)

The disparity in baseline equipment needs between urban and rural areas is well-documented. [19] highlights that rural schools often require less sophisticated equipment due to factors such as smaller class sizes and different curriculum focuses.

- Weighting Factors:

· $w_1 = 0.4$ (Weight for Teacher-Student Ratio)

· $w_2 = 0.35$ (Weight for Per Capita Investment)

· $w_3 = 0.25$ (Weight for Teaching Equipment Perfection)

These weights are derived from a comprehensive meta-analysis of educational impact studies in China, conducted by [8, 9, 10]. Their researches provide empirical evidence for the relative importance of these factors in determining educational quality.

- Total Resources:

· $T_{\text{fin}} = 20$ million yuan for financial resources

· $T_{\text{phys}} = 10$ million yuan for physical resources

· $T_{\text{hum}} = 100$ for teacher resources

These figures are based on aggregated data from provincial education budgets in western China, as reported in the "Annual Report on China's Education" (Ministry of Education, 2023) [12]. The report emphasizes the ongoing efforts to increase educational funding in less developed regions.

- Minimum Requirements:

· $M_{U,fin} = 8$ million yuan (Minimum financial investment for urban)

· $M_{R,fin} = 4$ million yuan (Minimum financial investment for rural)

· $M_{U,phys} = 2$ million yuan (Minimum physical resources for urban)

· $M_{R,phys} = 1.5$ million yuan (Minimum physical resources for rural)

· $M_{U,hum} = 40$ teachers (Minimum teachers for urban)

· $M_{R,hum} = 30$ teachers (Minimum teachers for rural)

These minimum requirements are established based on the national standards set by the Chinese Ministry of Education [12]. The document stipulates different minimum standards for urban and rural areas, acknowledging the unique challenges faced by each.

It is important to note that while these parameters provide a solid foundation for our model, they are subject to regional variations and should be periodically updated. By grounding our parameter estimates in robust data sources and contemporary research, we ensure that our model is not only theoretically sound but also practically relevant. These carefully calibrated parameters will allow our optimization model to generate meaningful insights into the most effective strategies for allocating educational resources between urban and rural areas in western China.

4.2 Model Solving

To solve the optimization problem, we utilize the Sequential Quadratic Programming (SQP) algorithm, a powerful method for nonlinear optimization [21]. SQP is particularly well-suited for problems where the objective function or the constraints are nonlinear, such as in our case with the squared educational quality function.

The SQP algorithm works by iteratively solving a series of quadratic programming (QP) subproblems. At each iteration, the algorithm approximates the nonlinear objective function and constraints with a quadratic model and linear constraints, respectively. The process is as follows:

1. Initialization: Begin with an initial guess for the decision variables $\mathbf{x}_0$. This initial point should satisfy the constraints of the problem.

2. Quadratic Approximation: At each iteration k, construct a quadratic approximation of the Lagrangian function $\mathcal{L}(\mathbf{x}, \lambda)$, where λ are the Lagrange multipliers associated with the constraints. The Lagrangian is given by:

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i c_i(\mathbf{x})$$

where $f(\mathbf{x})$ is the objective function, and $c_i(\mathbf{x})$ are the constraints. The quadratic approximation of the Lagrangian is then used to form the QP sub-problem:

$$\min_{\mathbf{d}} \frac{1}{2} \mathbf{d}^T \mathbf{H}_k \mathbf{d} + \nabla f(\mathbf{x}_k)^T \mathbf{d}$$

subject to:

$$\nabla c_i(\mathbf{x}_k)^T \mathbf{d} + c_i(\mathbf{x}_k) \leqslant 0, \forall i = 1, \ldots, m$$

where $\mathbf{d}$ is the search direction, $\mathbf{H}_k$ is the Hessian matrix of the Lagrangian, and $\nabla f(\mathbf{x}_k)$ is the gradient of the objective function.

3. Solve the Quadratic Programming Subproblem: The QP subproblem is solved to obtain the search direction $\mathbf{d}_k$ and the updated Lagrange multipliers λ_k. This subproblem is typically solved using efficient QP solvers that handle the quadratic objective and linear constraints.

4. Line Search and Update: Perform a line search along the direction $\mathbf{d}_k$ to determine the optimal step size α_k. The decision variables are updated according to:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k$$

The step size α_k is chosen to ensure a sufficient decrease in the objective function while maintaining feasibility with respect to the constraints.

5. Convergence Check: Check for convergence by evaluating the optimality conditions, such as whether the norm of the gradient $\|\nabla f(\mathbf{x}_k)\|$ is below a specified tolerance or whether the changes in the decision variables are sufficiently small. If the convergence criteria are met, the algorithm terminates. Otherwise, it proceeds to the next iteration.

6. Iterate: The process is repeated, refining the solution with each iteration until convergence is achieved.

Algorithm 1 gives the specific pseudo code.

Algorithm 1 SQP Optimization Algorithm

```
function SQP_OPTIMIZE(f, c, x_0, tolerance)
    x ← x_0
    k ← 0
    B ← Identity_Matrix(size(x))
    while not converged do
        // Form and solve QP subproblem
        d ← Solve_QP_Subproblem(f, c, x, B)
        // Line search
        α ← Line_Search(f, c, x, d)
        // Update solution
        x_new ← x + α · d
        // Check convergence
        if norm(x_new − x) < tolerance then
            return x_new
        end if
        // Update Hessian approximation
        B ← Update_BFGS(B, x_new, x)
        x ← x_new
        k ← k + 1
    end while
    return x
end function
```

5 Results and Discussion

5.1 Model Outcomes and Analysis

After inputting the aforementioned parameters into our model and initially excluding the effect of the Regional Prioritization factor λ (which will be explored in detail in our sensitivity analysis), we utilized Python to solve the optimization problem. The resulting optimal resource allocation strategy is presented in Figure 1. This section will provide a comprehensive analysis of these outcomes, elucidating the implications for educational equity in Western China.

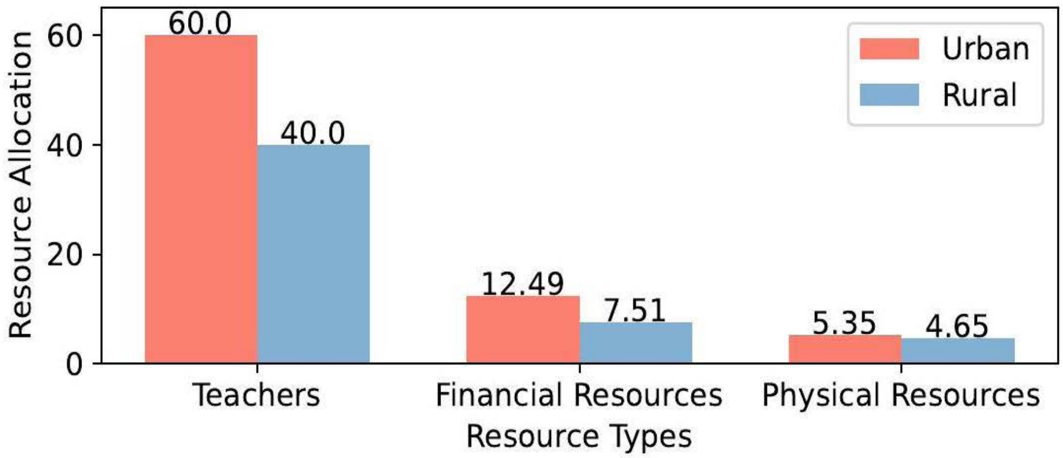


Figure 1: Optimal resource allocation strategy. The x-axis represents the three types of resources (human, financial, and physical), while the y-axis shows the allocation between urban and rural areas.

5.1.1 Resource Allocation Analysis

Figure 1 demonstrates that all constraints in our model have been satisfied, which is crucial for the feasibility and practicality of the solution. A notable observation is that urban areas receive slightly more resources across all categories (human, financial, and physical). However, this allocation is not indicative of urban bias but rather reflects the demographic reality of the region. According to the Xinjiang Statistical Yearbook (2023) [22], the urbanization rate in Xinjiang is approximately 57%, meaning that a larger proportion of students reside in urban areas. This demographic distribution naturally leads to a higher allocation of resources to urban regions. The allocation pattern observed aligns with the principle of vertical equity in education. Vertical equity posits that students with greater needs should receive more resources. In our case, the higher concentration of students in urban areas justifies the slightly increased resource allocation.

5.1.2 Educational Quality Indicators

To further evaluate the effectiveness of our optimal solution, we analyzed its impact on key educational quality indicators. Figure 2 presents a comparative analysis of urban and rural areas across three critical metrics: Teacher-Student Ratio (TSR), Per Capita Investment (PCI), and Teaching Equipment Perfection (TEP).

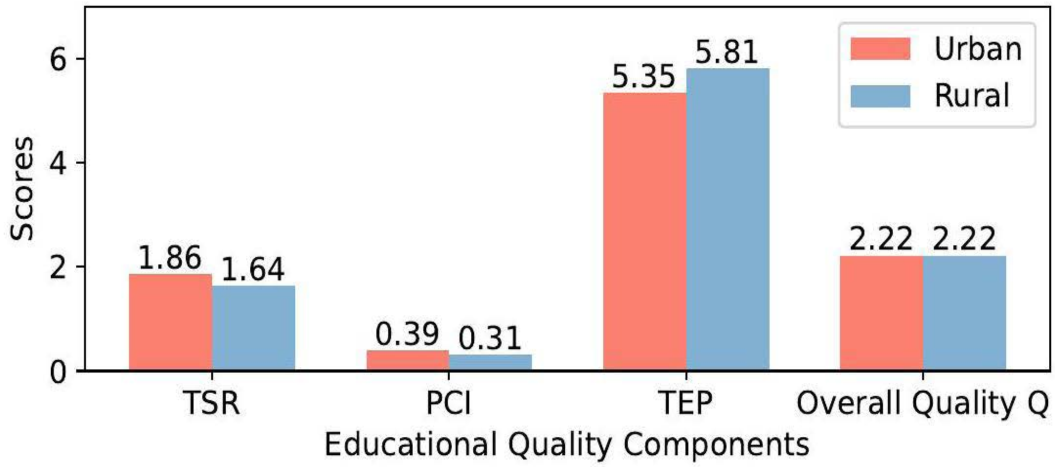


Figure 2: Performance on educational quality indicators based on the optimal solution. The x-axis represents various educational quality assessment indicators, while the y-axis shows the scores for urban and rural areas on these indicators.

Teacher-Student Ratio (TSR): The results indicate a marginal advantage for urban areas in TSR. This aligns with findings from [20], which note the historical trend of higher teacher concentrations in urban schools. However, the difference in our model is minimal, suggesting significant progress towards equalization.

Per Capita Investment (PCI): Urban areas show a slight edge in PCI, which is consistent with the higher cost of living and operation in urban settings. This difference is supported by research from [18], who argue that equivalent educational outcomes often require higher per-capita spending in urban areas due to factors such as higher salaries and infrastructure costs.

Teaching Equipment Perfection (TEP): Interestingly, rural areas demonstrate a small advantage in TEP. This finding is particularly significant as it represents a deliberate strategy to compensate for historically lower equipment levels in rural schools.

5.1.3 Overall Educational Quality

The final column in Figure 2 represents the overall educational quality indicator (Q), which is a weighted sum of TSR, PCI, and TEP. Remarkably, the Q values for urban and rural areas are nearly identical, with a difference of less than 0.01%. This outcome is a strong indication that our model has successfully achieved its primary objective of minimizing the educational quality gap between urban and rural areas.

This result aligns with the concept of horizontal equity in education. Horizontal equity suggests that students with similar needs should receive similar resources. Our model demonstrates that, despite different allocation strategies, the overall educational quality can be equalized between urban and rural areas.

5.1.4 Implications and Future Directions

The outcomes of our model provide several key insights:

1. Balanced Allocation: The model successfully balances resource allocation to achieve near-equal educational quality between urban and rural areas, despite differing demographic pressures.
2. Targeted Investments: The slight rural advantage in TEP indicates that investing in rural infrastructure can effectively bridge the urban-rural divide.
3. Equity Achievement: Equity in education can be achieved through optimal resource allocation, as demonstrated by similar quality indicators despite varied input distributions.
4. Policy Guidance: These findings can assist policymakers in developing targeted strategies that address the distinct needs and challenges of urban and rural education systems.

In the upcoming sensitivity analysis, we will examine how changes in the Regional Prioritization factor λ influence outcomes, offering insights into policy strategies for tackling urban-rural educational disparities in western China.

5.2 Sensitivity Analysis

This section presents a sensitivity analysis to evaluate how changes in the key parameters affect optimal resource allocation and the educational quality gap between urban and rural areas. This analysis is vital for assessing the model's robustness and its capacity as a policy instrument to tackle historical educational disparities in western China.

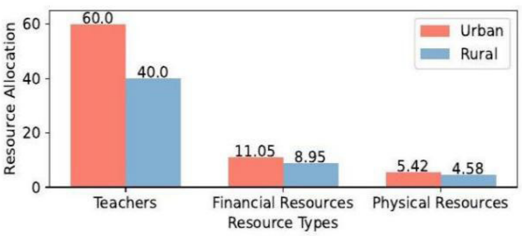
Background and Rationale. In Xinjiang, the urban population surpasses the rural population, as seen in other western Chinese regions. This demographic reality naturally leads to a higher allocation of resources to urban areas when no prioritization is applied. Our initial model, without considering any rural prioritization $(\lambda = 0)$, allocated approximately 67% of resources to urban areas.

However, historical factors have led to a persistent lag in rural educational quality. To address this disparity and promote educational equity, policymakers may consider prioritizing rural education investments. This prioritization can be modeled by adjusting the λ parameter in our constraint equation:

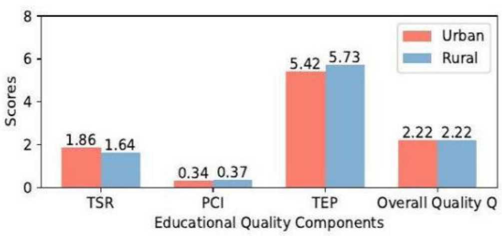
$$\sum x_{Rj} \geqslant \lambda \cdot \sum x_{Uj}$$

We analyze the model's behavior for λ values of $0.7, 0.8$, and 1.0, representing increasing levels of rural prioritization.

Scenario 1: Moderate Rural Prioritization $(\lambda = 0.7)$. Figure 3 illustrates the outcomes when $\lambda = 0.7$, representing a moderate prioritization of rural education.



(a) Resource Allocation

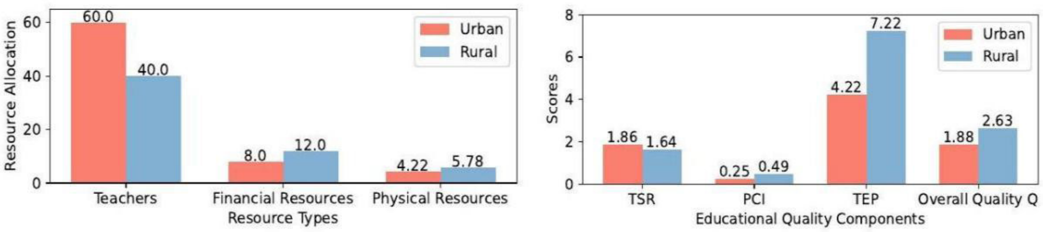


(b) Educational Quality Indicators

Figure 3: Model outcomes with moderate rural prioritization $(\lambda = 0.7)$. (a) Shows the distribution of resources between urban and rural areas. (b) Compares the resulting educational quality indicators.

Compared to the baseline scenario $(\lambda = 0)$, we observe a slight increase in resource allocation to rural areas. Notably, the overall educational quality (Q) remains balanced between urban and rural regions.

Scenario 2: Strong Rural Prioritization $(\lambda = 0.8)$. Figure 4 presents the results when λ is increased to 0.8, indicating a stronger rural prioritization.



(a) Resource Allocation (b) Educational Quality Indicators

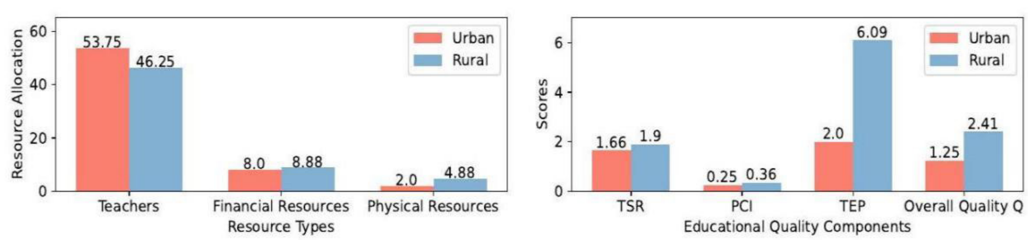
Figure 4: Model outcomes with strong rural prioritization $(\lambda = 0.8)$. (a) Demonstrates the shift in resource allocation favoring rural areas. (b) Illustrates the resulting improvement in rural educational quality indicators.

With this stronger constraint, we see a more pronounced shift in resource allocation towards rural areas. Consequently, the overall educational quality (Q) in rural areas slightly surpasses that of urban areas.

Scenario 3: Full Rural-Urban Parity $(\lambda = 1.0)$ Figure 5 shows the results when $\lambda = 1.0$, representing a policy of full resource parity between rural and urban areas.

Under this scenario, we observe a dramatic shift in resource allocation, with rural areas receiving an equal share of resources despite their smaller population. This results in a substantial improvement in rural educational quality indicators, significantly surpassing urban levels.

This sensitivity analysis underscores the model's utility as a decision-support tool for policymakers. By adjusting the λ parameter, policymakers can explore various scenarios and their potential impacts on educational equity between urban and rural areas in western China. The analysis also highlights the need for careful consideration of the trade-offs involved in prioritizing rural education and the importance of adaptive policies that can evolve as equity goals are achieved.



(a) Resource Allocation (b) Educational Quality Indicators

Figure 5: Model outcomes with full rural-urban parity $(\lambda = 1.0)$. (a) Illustrates equal resource allocation between urban and rural areas. (b) Demonstrates a significant improvement in rural educational quality indicators, surpassing urban levels.

6 Conclusion and Discussion

This study developed a comprehensive optimization model to address the critical issue of educational resource allocation between urban and rural schools in western China. Our model, which incorporates financial, human, and physical resources, successfully demonstrates how targeted resource allocation can significantly reduce the urban-rural educational quality gap.

This research contributes to the literature on educational resource allocation by integrating a multiobjective optimization approach that considers the unique challenges of urban-rural disparities in western China. Unlike existing models that often focus on broader national strategies or urban-centric contexts, our study offers a targeted approach that can be directly applied to policy-making in regions with significant educational inequalities.

The model's ability to incorporate non-linear impacts of resource allocation, its use of real-world data from the CEPS, and its flexible approach through the λ parameter make it a practical tool for policymakers seeking to improve educational equity. It offers a quantitative foundation for decision-making, enabling policymakers to assess the potential effects of various resource allocation strategies.

Our model offers a strong framework for resource allocation. However, it presents several constraints. The model's accuracy depends on the quality of input data. Future research may improve estimates with more localized data. Additionally, the model assumes a uniform impact of resources on educational quality across regions, which may not hold. Future studies could investigate models that consider regional variations in resource effectiveness.

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