Analysis of the Trajectory of Free-throws

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Abstract

This research paper delves into the intricate realm of free-throw shooting in basketball, combining advanced data analytics and artificial intelligence to dissect the trajectory of human-generated free-throws and induce the theoretically "perfect shot" by AI. This study aims to introduce the sport of basketball, provide valuable insights into the science of basketball shooting, and ultimately contribute to developing the skill of free-throw shooting. I used a real-life example of a free-throw shot to achieve this goal. Using the model, I molded a horizontal and vertical distance-time graph on the coordinate plane to estimate the velocity at each second. By utilizing some physics kinematics, I was able to draw a free-body diagram of the basketball in motion and analyze the possible factors affecting its trajectory. Finally, as all my research points toward the same goal, I hope to mathematically optimize the perfect trajectory to maximize the likelihood of a perfect shot by an AI.

Keywords: basketball, Free-throws, force analysis

1. Introduction

Basketball is one of the most popular sports in modern society. As school urges students to participate more in outdoor activities and physical exercises, many people choose the sport basketball. However, like all sports, it takes a lot of practice and analysis to fully understand it. In my research paper, I will analyze a basketball's trajectory and determine factors that affect it.

As most people know, the free-throw shot is the most essential part of the sport. I practice my free-throw shooting daily because it is a crucial game aspect. I want to perfect my free-throw shooting by analyzing its trajectory. Unlike three-point shots, free-throw is relatively closer to the basketball hoop (approximately 4.57 meters) and is one of the best positions to practice shooting. Beginners tend to face the problem of shooting in the wrong direction or angle with destructive power. At 4.57 meters in front of the basketball rim, the free-throw line is parallel to the basketball rim. To successfully throw the ball into the rim, you must aim using the correct angle and power. However, basketball has no fixed angle because you can make shots at all angles with a corresponding capacity to it. Because of that, I will research the correlation between the tip of your shot and the velocity it should have. Furthermore, I will point out some factors that could affect our shooting.

2. Context

2.1 Data of myself shooting a free-throw

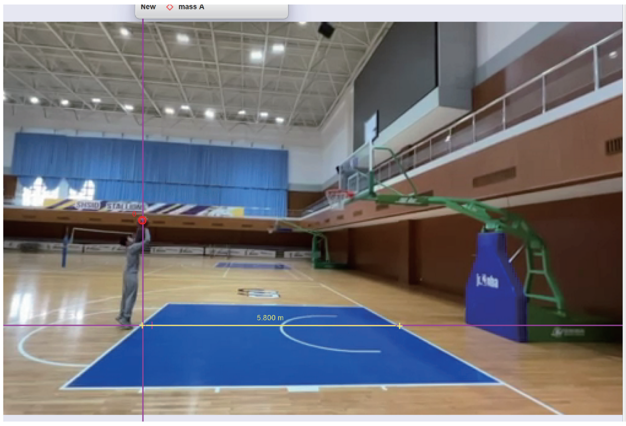
First, I recorded a video of me shooting a free throw shot. The location is at our school's indoor basketball gym. The rim's size and height are the same as the professional FIBA rim, with a diameter of 46cm and a height of 3.05m, respectively. The distance from the free-throw line to the rim is 4.57m.

Next, I will find the general equation of my ball's trajectory. To do so, I used the Tracker app and tracked the movement of my ball on a set of coordinates. I set my position as the origin, with the free-throw line as the x-axis and the line between me and the rim as the y-axis. In real life, these two lines are perpendicular bisectors to each other, but the picture may look different because of angle issues. Then, I tracked the location of my basketball at each frame in the coordinate plane. There were 98 frames, beginning before the ball left my hands to the moment the ball went into the hoop. The trajectory of my ball is not perfect because of factors like wind, but I found a general curve.

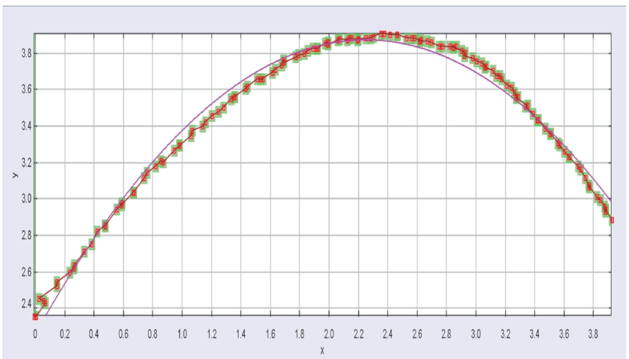
t(s)	x(m)	y (m)
0.000	-9.576E-3	2.359
0.033	5.978E-3	2.359
0.067	6.820E-2	2.429
0.100	5.653E-2	2.441
0.133	2.542E-2	2.456
0.167	0.146	2.526
0.200	0.146	2.546
0.233	0.235	2.600
0.267	0.263	2.623
0.300	0.267	2.639
0.333	0.333	2.713
0.367	0.383	2.752
0.400	0.418	2.822

t(s)	x(m)	y (m)
0.433	0.473	2.857
0.467	0.477	2.853
0.500	0.554	2.946
0.533	0.589	2.977
0.567	0.585	2.970
0.600	0.667	3.040
0.633	0.667	3.040
0.667	0.741	3.117
0.700	0.760	3.145

Some of the points I tracked

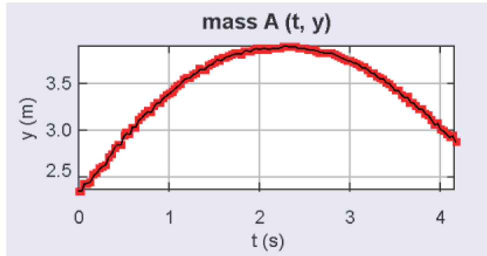


The coordinate plane and the starting coordinate of my curve



The curve formed from connecting all the points

In this graph, the parabola generally covers all the points that the Tracker marks. Using these points, I found the function of this parabola in terms of x (the horizontal distance from the free-throw line to the rim) and y (the height of the ball) as time progresses using regression. Since this curve is a parabola, I can view it as a quadratic function f(x)=Ax^2+Bx+C. Here is my function: f(x)=-(3.184*10^-1)x^2+1.433x+2.263. This parabola is very similar to the trajectory of my ball. However, to fully analyze the components driving the ball, I must separate the function in terms of horizontal and vertical velocity. This is because the ball moves toward the rim at a specific angle (not equal to 45 degrees). Therefore, its vertical and horizontal distances are different, causing a difference in velocity. (v=d/t).

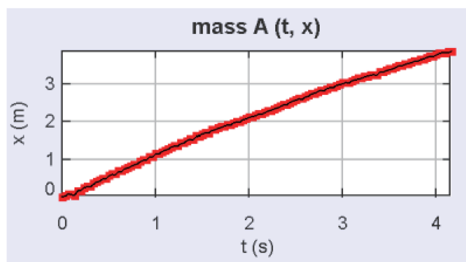


The time over vertical distance graph. The slope at each point is its velocity.

2.2 Analysis of the trajectory of the basketball

First, we analyze the vertical velocity of the ball. Since velocity equals distance divided by time, we can see the change in velocity by making a d-t graph. The x-axis is time, and the y-axis is the ball's vertical distance. As we can see, this curve is similar to the previous angle between vertical distance and horizontal distance. Despite the x-axis changing from horizontal distance to time, the height of the ball still follows the same pattern. It first increases and reaches a maximum before returning to its original position. Why though? This is because the horizontal distance is directly proportional to time. In other words, as time increases, the horizontal distance increases proportionally. As a result, switching them in the coordinate plane will not change the parabola.

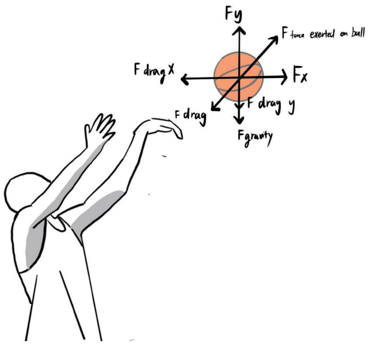
Now, let's talk about the horizontal velocity of the ball. As said in the previous paragraph, the horizontal distance is proportional to time. It is because as time increases, the ball will continue to move forward until it stops. As a result, instead of being a parabola, the equation for this graph is a linear line: y=9.49t+0.14



The time over horizontal distance graph. The slope at each point is its velocity.

2.3 Analysis of the Factors Affecting Shooting

Now, we are done analyzing the vertical and horizontal velocity of the ball. Next, we should analyze its correlation to our launching angle(the angle at which we shoot the basketball) and other factors that can affect the ball's velocity.



The force analysis of the basketball we shoot

As we can see in the graph above, five forces are acting on the ball. (There may be other forces, but we can neglect them because of their small magnitude.) The force we work on the basketball is at a specific degree θ, which can be separated into vertical and horizontal components. Air resistance or Fdrag is in the opposite direction, with an angle of θ, which can also be divided into vertical and horizontal components. The force acting in the x-axis is Fx and Fdragx (the horizontal component of air resistance). Fx, on the graph, can be written as F(force exerted on the ball)*cosθ. Fdragx can be written as Fdrag*cosθ. Typically, Fdrag for a basketball is approximately 10% of gravity. To calculate the net force of the ball in the x-axis, we subtract Fx with Fdragx. Fnet=F*cosθ-Fdrag*cosθ. Since Fdrag is only 10% of gravity, it is obvious that Fx is larger than Fdragx. As a result, the net force of the basketball's horizontal component is positive (to the right). That is why the horizontal distance-time graph is linear, increasing at the same rate.

For the vertical component of the ball, three forces are acting on it: Fy, Fdragy, and Fgravity. Since Fy is acting upwards and Fdragy and Fgravity are acting downwards, we need to subtract Fy from Fdragy and Fgravity to find the net force. Fy=F*sinθ, Fdragy=Fdrag*sinθ, and Fgravity=mg. At the start, Fy is larger than the sum of Fgravity and Fdragy, so the velocity is positive. Then, the ball starts to slow down until the velocity is negative. (Ffragy and Fgravity are larger than Fy) Unlike velocity in the horizontal component, which is constant, velocity in the vertical component changes because of factors like gravity and air resistance. That is why the slope of the graph is changing, thus making the function a parabola. (The slope of a point is the slope of the tangent line to the point.)

3. Conclusion

In conclusion, this research paper has explored the intriguing world of free-throw shooting in basketball. In an era when artificial intelligence is popular, artificial intelligence can be used to evaluate basketball. A few years ago, alphago proved that AI had surpassed humans in the field of Go. If AI technology is applied to basketball, AI may be able to use algorithms to help humans calculate how to shoot more accurately. Therefore, future research can focus on how to combine AI and basketball fields.

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