Research on Opinion Dynamics on Social Networks

Ke Wu

Abstract

Opinion dynamics, which focuses on the formation, propagation, and evolution of people's opinions in social networks, has recently been widely studied. This paper outlines the history of opinion dynamics research, current methods, and future directions. Specifically, some basic theoretical concepts of graph theory are initially introduced. Then, the key research findings of opinion dynamics, such as Voter's and Degroot's models, which are discrete and continuous models, are reviewed. Finally, the future direction of this field is discussed.

Keywords: social network, continuous model, discrete model, opinion dynamics

1 Introduction

Nowadays, people's social lives and information sharing have significantly changed due to the quick growth and widespread use of social media platforms. Social media like Facebook, Twitter, and Sina Weibo have developed into significant venues for individuals to express their ideas and get feedback. This allows the quick and widespread distribution of ideas and encourages exchanges and discussions among people. Opinion dynamics is a term used to describe this opinion exchange and dissemination process.

The primary area of research in opinion dynamics focuses on how opinions emerge, change, and disseminate via social networking sites. Sharing information among people on social networking sites is a critical component. The regulatory mechanism and evolutionary law of opinion transmission on social media platforms are revealed through the research of opinion dynamics. Many elements, including social network structure, information transmission channels, and individual qualities, impact how people create opinions. Additionally, on social media sites, people's ideas propagate to others and change over time.

The study of opinion dynamics focuses on the effect of individual opinions on areas including politics, economics, culture, and social networks. Opinion dynamics can help us understand how political beliefs are formed and propagated. Opinion sharing on social media can affect voter behavior, election outcomes, and policy-making[1]. In economics, opinion dynamics can help economists comprehend how consumers' perceptions are influenced and how goods are pushed[2]. By examining perspective dynamics, we may get insight into how opinions on social networks influence customer preferences and brand loyalty, guiding business marketing efforts. In culture, opinion dynamics help us better understand how cultural beliefs are passed down and developed through time. By analyzing opinion dynamics, we may learn how social network opinions affect how cultural creations are shared and received, as well as how this affects cultural values are formed and passed down[3].

This paper will give a theoretical investigation into the opinion dynamics in social networks. First, the paper will provide a basic understanding of opinion dynamics research methodologies and concepts. The relevant background information, several classical opinion dynamics models, and the most recent opinion dynamics research findings will then be presented. The summary and prognosis section will provide readers with a last perspective on future study paths. This paper aims to further the research on opinion dynamics on social networks by serving as a valuable resource for scholars and decision-makers.

2 Theoretical Foundations

Typically, a social network containing individuals can be described as an n degree-weighted directed graph, where $g[A]=(V,\epsilon[A],A)$ has the set of nodes is $V=\{v_1,...v_n\}$ and the coefficients are $I=\{1,...n\}$. An edge $e_{ij}=(v_i,v_j)$ is contained in the set of ordered edges $\epsilon[A]\subseteq V^2$ if and only if$a_{ij}>0$. If node i has $a_{ii}>0$; then it has a self-loop. When the arrow points from v_i to v_j, at that point, the edge e_{ij} is considered to be the in-degree *concerning*v_j, and the out-degree co*ncerning*v_i, $a_{ji}$is considered to be the weights connected to the edgee_{ij}.

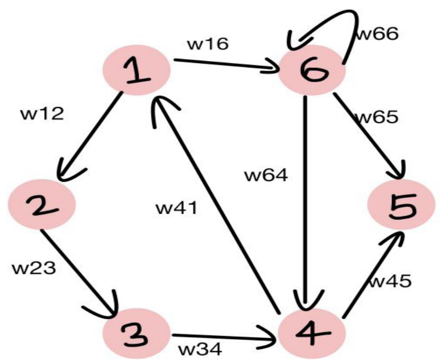

Figure 1. An example of a six-person social network

An example of a six-person social network created to resemble a directed graph$g[A]$ is shown in Figure 2.1. People may think of the edges' orientations as influencing directions. For instance, utilizing directed edges$e_{12}$and influence weightsw_{12}, Individual 1 persuades Individual 2 to alter their attitude. Through a directed path made up of several edgese_{12}, e_{23}, e_{34}, e_{45} with weights ofw_{12}, w_{23}, w_{34}, w_{45}, Individual 1 indirectly impacts Individual 5. Individual 6 has a self-loope_{66}.

An edge's significance varies depending on the specific model and its scope, but it often denotes some sort of direct contact between two individuals,i and j. The edge\[{}_{}\]in the opinion models considered in this study denotes that person j is aware of the person*I*view and may be affected by it. In the models cited in this paper, only positive edge weights graphs, i.e., those individuals $a_{ij} \geq 0$,will be considered. Negative edge weights may be interpreted as a surrogate for unfriendly or conflicting interpersonal impacts by other models.

Since the graph is directed, it cannot be assumed that A is symmetric since the presence of e_{ij} does not necessarily entail the presence\[{}_{}\]of eji and vice versa. The nodes*j*that record the in-degree of an edge v_i in the set of neighbors are denoted by $N_1 = \{v_i \in V : (v_i, v_i) \in \epsilon\}$. $v_i$may be included in the set N_1since self-loops are permitted. A directed path is a column of edges $(v_{p_1}, v_{p_2}), (v_{p_2}, v_{p_3}), \ldots$ where $v_{p_i} \in V, e_{p_i p_{i+1}} \in \epsilon$. If there is a directed path from v_j tov_i, node*j*can reach another node*i*. Only when there is a direct connection linking every node in a graph$g[A]$can it be called to be highly linked. A directed cycle is a route with just the beginning (and ending) vertex as a repeating node that starts and finishes at the same vertex. The cycle length is a function of the number of edges in a cyclic path. The minimal value that cuts each cycle in half in a directed graph shows that the graph is periodic. The graph isn't periodic if and only if$k = 1$. Self-loops never have periodicity.

Based on the discovery of nonnegative matrices and the Perron-Frobenius theorem, A contains u^T and v left and right eigenvectors with nonnegative nonnegative elements associated with eigenvalues $\lambda_1 = \rho(A)$. If $g[A]$ are strongly connected, it can be assumed thatu^T and v have positive definite incidence, and $\lambda_1 = \rho(A)$ is a simple eigenvalue. The rest of the eigenvalues λ_i and the eigenvalues $i \neq 1$ that satisfy $|\lambda_i| = \rho(A)$ are simple values[4].

2.2 Network Topologies

2.2.1 Regular Network

Regular networks are frequently used to explain and simulate people's actions and decisions in the process of information transmission and perspective creation in the study of opinion dynamics. A regular network has a defined topology, with nodes and edges connected in a way that complies with predetermined criteria. Different regular networks can successfully affect the result of information transmission and opinion formation by forming, matching, enforcing, and developing.

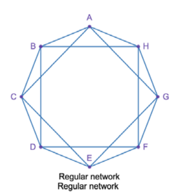

Figure 2. Regular Network

(1) Fully Connected Network

A fully Connected Network is one in which all nodes are directionally connected. Each node in a network with n

nodes will have $n-1$ edges connecting it to other nodes. However, Fully Connected Networks are uncommon in real-world applications due to their complexity [5].

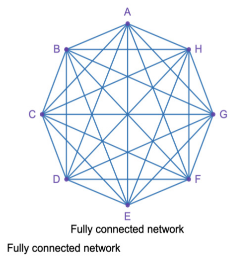

Figure 3. Fully Connected Network

(2) Grid Networks

A network topology that resembles a flat grid is called Grid Networks. It is made up of several nodes, each of which stands in for a distinct person and connects to make the grid[6]. For instance, each node in a two-dimensional mesh network is typically connected to its top, bottom, left, and right neighbors, except boundary nodes. This pattern is commonly utilized in different disciplines, such as lattice structures in physics[7], GIS maps[8], and pixel grids in computer science[9].

(3) Cyclic Networks

Cyclic Networks are a unique network structure comprising several nodes linked to one another in a ring pattern[10]. A closed ring structure is created by connecting each node directly to its neighbors. Additionally, the first and end nodes are adjacent, and this connection creates a loop. Applications for biological neural networks are numerous[11].

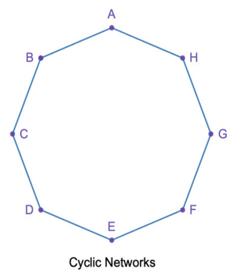

Figure 4. Cyclic Network

2.2.2 Generative Network

Unlike regular networks, generative networks have a non-uniform network topology where most nodes only have moderate connectedness, and only a few have extremely high connectivity. Generative networks have scale-free qualities in the network structure and are created by a generative mechanism (such as a stochastic or an optimization process). Indicating that the degree distribution of the nodes follows a power law distribution rather than a typical random distribution like the Poisson distribution[12].

(1) Small-World Network

A network with a high clustering coefficient and a brief average path length is called a small-world network. While a low average path length suggests that node distances are often minimal, a high clustering coefficient suggests that the nodes in the network prefer to form close-knit clusters. Additionally, small-world networks are a hybrid between regular and generative networks, meaning that both regular and random connections exist between nodes[13]. Numerous social and natural systems, including human social networks[14] and actor cooperative networks[15], exhibit this network topology.

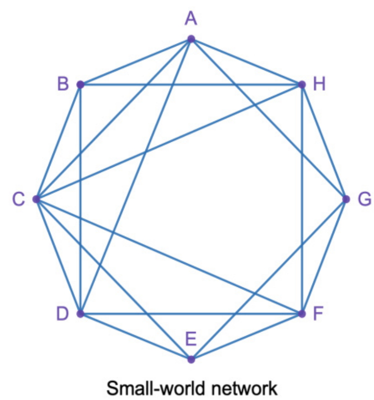

Figure 5. Small-world Network

(2) Scale-free Network

A scale-free network is a model with a power law distribution of node degrees. This network is characterized by a few nodes with extremely high degrees, while most have relatively low degrees. These highly connected nodes play a key role in the network and are known as center or hub nodes[16]. Scale-free networks have been widely studied and applied in many fields, such as the Internet[17] and biological networks[18].

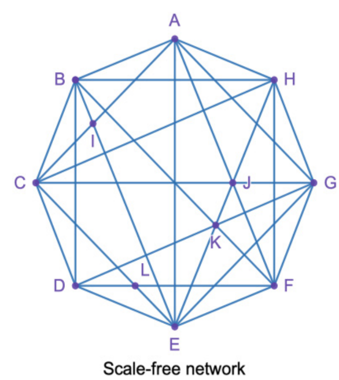

Figure 6. Scale-free Network

(3) Random Networks
In Random Networks, connections between nodes are created entirely at random. Consider the Erdos-Renyi model, a well-known example of a random network model in which connections between each pair of nodes have the same probability[19].

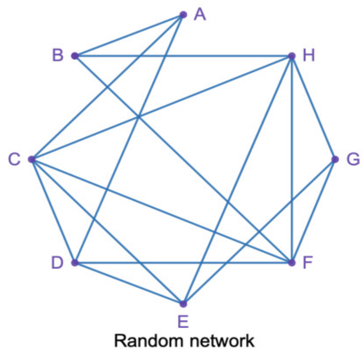

Figure 7. Random Network

2.2.3 Real Social Networks

Scale-free and small-world features are frequently seen in social networks like Twitter and Facebook. As an illustration, Twitter is a typical social networking site where users may follow one another and post 140-character tweets. Additionally, the Twitter network is made up of people who communicate with one another by following one another.
The Twitter network demonstrates scale-free features, i.e., a few highly concentrated nodes with many followers, such as celebrities or news organizations. At the same time, a sizable population of common users with few followers exists. The Twitter network also exhibits a small-world property, where the distance between users is typically quite tiny. This is due to the ease with which users may locate other users who share their interests or viewpoints. A user can get information uploaded by another celebrity through the following link of, say, a celebrity who follows another celebrity.

3 Classical Models

3.1 Discrete models

We may distinguish between discrete and continuous perspective dynamics models based on how views are represented. Views are represented as discrete numerical variables in discrete models. The voter model, the majority adjudication model, and the Sznajd model are classic discrete models. Research on discrete models has garnered a lot of attention, and a lot of useful results have been found.
According to the discrete model, each social network member has just one of two possible viewpoints, symbolized by the numbers 0 and 1, respectively. When two connected people have opposing viewpoints, the possibility that they will not remain connected is α, and the other person will either (a) form a new connection with someone who shares their viewpoint or (b) form a new connection with a randomly chosen person from the entire network. If this occurs, the probability that one will mimic the other person's viewpoint is $1-\alpha$[20].

3.1.1 Voter model

The Voter model, one of the most thoroughly studied models of opinion dynamics, was one of the earliest models of opinion dispersion to be put out. The voter model is developed based on the idea of persuasion, which is an intuitive phenomenon in opinion distribution. Clifford and Sudbury initially discussed the idea of voter dynamics in their 1973 paper on species competition[21], while Holley and Liggett referred to the model as the voter model in 1975 first[22].
The voter model is based on the Ising model spin theory, which aims to consider the dynamics of views in the context of a network characterized by the voter model and has N nodes and$L$ linkages. Every node in the graph stands for a voter, and each voter can endorse either a good view (referred to as $+1$) or a negative opinion (referred to -1). The network displays the voter's selection as a spin, which may be upward or downward spinning. Each voter's viewpoint is assumed to be represented by the spin variable$\sigma_i$on their lattice:$\sigma_i = \pm 1$.
The following measures may be taken to update the network's nodes. A node in the network is chosen at random, and the spin value of that node is then modified using a simple majority voting process. After scaling with the spin values of its neighbors, the node's spin value, $ie.k_+ / (k_+ + k_-)$, which has a normal distribution, is obtained, where the number of neighbors' nodes$k_\pm$with spin values of ± 1, is shown. Therefore, regardless of the focused node's prior state, the possibility that the next picked node will have a positive spin is higher if the focal node's positive spins are greater than the negative spins. The node's future state, as determined by this update rule, is unrelated to its initial state[23].
Voter models offer a wide variety of practical uses. For instance, academics have created various voter models for the US elections. The introduction of these models has resulted in the stabilization of electoral circumstances like "consensus" when voters agree or "clustering" when opinions are distributed in confined places[24].

3.1.2 Majority Adjudication Model

In contrast to the voter model, the majority adjudication model is based on observing various civilizations. Human and animal civilizations both contain members from several social strata. In an animal ecosystem, we can divide animals into predators and prey. So, we can divide people into leaders and ordinaries in human society. “Leaders” might be politicians, music stars, performers in movies, religious figures, etc., and”ordinary people” may be the others [25].

We utilize the spin value to describe whether individuals are in favor of or against a social issue in the majority adjudication paradigm. In the model, we also created two personas: $\tau$for the “leader” type and$\sigma$for the “guided” kind. We give each lattice a spin variable $\sigma_i = \pm 1$. At each step, we choose one spin and work out the signs of most of the spins immediately surrounding it. We apply the relevant sign to the chosen spins based on the probability of P_{agree} . When noise parameter $q = 1 - P_{agree}$, the spin we choose will have the opposite sign. The following equation can be used to determine the likelihood of the flip:

$$w(\sigma_i) = \frac{1}{2}\left[1 - (1-2q)\sigma_i S\left(\sum_{\delta=1}^{z}\sigma_{i+\delta}\right)\right] \text{(Eq. 1)}$$

For any x and $x \neq 0, S(x) = sgn(x), S(0) = 0$. In a square regular lattice, each position is surrounded by four nearest neighbors, and the set of these neighbors is denoted by the symbol δ for z, the number of collocations. The probability function (1) that we define exhibits symmetry, e.g., the Ising model at the time of transformation also changes σ spins state with a probability of change $w(\sigma_i) = w(-\sigma_i)$ (Eq. 2)[26].

3.1.3 Sznajd Model

According to the voter model, a person generally changes their mind after being persuaded by neighbors, shown by the neighbor’s viewpoint being given more weight. However, as the proverb goes, “Three people make a tiger,” since numerous people have a larger power to convince, the impact of many people is greater than that of a single person throughout the persuasion process. This concept forms the basis of the persuader model. People will more likely accept the majority opinion of their immediate neighbors as their own since most people who share the same perspective are more persuasive. This concept served as the foundation for Sznajd-Weron and Sznajd’s traditional Sznajd model, commonly called the “missionary model .”According to this paradigm, it is the duty of nearby “missionaries” to persuade their neighbors to share the same ideas or beliefs when they do so, and they cannot do so when they do not.

Since Józef Sznajd first introduced the persuader paradigm, it has primarily been applied online[27]. The model’s derivation is as follows:

1. At each time step, choose a pair of S_i and $S_{i+1} spins and$ use them to change their nearest neighbors. For example, choose the spins S_{i-1} and S_{i+2}
2. If $S_i = S_{i+1}$, then $S_{i-1} = S_i$ and $S_{i+2} = S_i$
3. If $S_i = -S_{i+1}$, then $S_{i-1} = S_{i+1}$ and $S_{i+2} = S_i$

According to the study’s findings, the voter model in the preceding section and the persuader model have certain parallels and contrasts in one-dimensional space. According to the experimental findings, the one-dimensional USDF model’s (i.e., the persuader model that underwent improvement) rules are nearly identical to those of the linear voter model. Katarzyna Sznajd Weron discovered the asymmetric coexistence of several views in contrast to the original USDF model. Stauffer’s research indicated that simultaneous updating makes it more challenging to maintain full consistency. This is because, during concurrent updating, certain decision-makers may acquire contradictory information (also known as frustration) from several neighbors yet still maintain their current opinions[28]. The Sznajd model, which Dietrich Stauffer subsequently dubbed, has been enhanced and applied broadly in a variety of industries, including marketing[29–31], banking[32], and politics[33–40].

3.2 Continuous models

A model type other than discrete is the continuous model, representing continuous variables across an interval. This group of models primarily consists of the Deffuant-Weisbuch (DW), Hegselmann-Krause (HK), Friedkin-Johnsen (FJ), and Degroot models, as well as several derivative versions and extensions of the model.

3.2.1 Deffuant-Weisbuch Model

The Deffuant-Weisbuch (DW) model is viewed as an interaction model within a continuous perspective model. The bounds of trust are constrained in this paradigm, meaning that perspective interaction is only feasible when the differences between two people’s viewpoints fall within a predetermined range[41]. An individual will interact with another person at some point, using the total difference between its perspective value and that of its neighbor as the viewpoint value at the next instant. Peer-to-peer communication, as opposed to group communication, is the norm among users of social networks. Based on the Deffaunt model, established research often examines the development of individual viewpoints[42–44].

Let the present opinions of individuals$i$and $j$who can be contacted be $op_i(t)$ and $op_j(t)$, respectively. After contact, the individual i's opinion is now updated according to $op_i(t+1) = op_i(t) + \mu[op_j(t) - op_i(t)]$ (Eq. 3), where the convergence parameter μ both indicates the speed of viewpoint update and the degree of confidence the individual i has in the surrounding individual j[45].

3.2.2 Hegselmann-Krause Model

When just certain users' effect on themselves is considered in updating user viewpoints, the HK model is suggested as a traditional continuous perspective interaction model. The model introduces the idea of a trust threshold by defining the difference in a certain range of the impact between user views and their own views. This trust level is represented by ϵ.

The repeating process of perspective exchanges in discrete time $T = \{0,1,2,...\}$ is simulated in the standard HK model by considering a population of N people. Each person at t time has a perspective attribute value $O_i(i = 1,2,...,N)$ that falls between. The average of the opinions of all the persons within range $[0,1]$ determines the individual i's perspective at $t+1$ time. The guideline for updating against points of view is:

$$O_i(t+1) = |I(i,O(t))|^{-1} \sum_{j \in (i,O(t))} O_j(t) \text{(Eq. 4)},$$

The individual i's finite trust set $I(i,O(t)) = \{1 \le j \le n \backslash |O_i(t) - O_j(t)| \le \epsilon\}$, which is made up of people who share an opinion with i and difference is smaller than or equal to ϵ. There are trust sets $|I(i,O(t))|$, and the weight of each person's effect on that i's perspective inside the trust set is $|I(i,O(t))|^{-1}$[46].

3.2.3 Degroot model

We suppose that a person A_i's opinion at any one time is $x_i(t)$ and that this viewpoint impacts A_j's opinion; this impact is w_{ij}, and $w_{ij} \ge 0$, $\sum_{j=1}^{n} w_{ij} = 1$. As a result, the method of fusing different perspectives may now be described as:

$$x_i(t+1) = w_{i1}x_1(t) + w_{i2}x_2(t) + ... + w_{in}x_n(t), t = 0,1,2,...(Eq.5)$$

Even though the original state is different, if everyone finally comes to an understanding in $\lim_{t\to\infty} x_i(t) = c(i = 1,2,3,....,n) \forall X(0) \in R^n$, we may claim that the group has reached a consensus. However, we can speak of polarization or division of the group's views when the group develops two or more distinct viewpoints during the stabilization period.

Eq. (5) expressed in terms of a matrix:

$$X(t+1) = W \times X(t), t = 0,1,2,...(Eq.6)$$

In Eq. (6), this is the DeGroot model when W is unaffected by passing time t or shifting public opinion X. The continuity of individual opinions $x(t)$ is typically presupposed $x(t) \in R$ in this approach. The DeGroot model shows that when a group comes to a consensus view, the opinion is a linear combination of all individual starting opinions, and the combination coefficient is connected to an eigenvector with an eigenvalue of 1[47].

In DeGroot's approach, weighing an individual's opinions with those of their neighbors is codified as the process of opinion creation. Each participant $i \in V$ in the model changes their opinion $s_i(t+1)$ over time $t+1$ as a weighted average of their view (which has weight w_{ii}) and the opinion of their neighbor (which has weight w_{ij} for the neighbor j). Remember that the values of w_{ii} represent the node's beliefs on its viewpoint and are independent of all other w_{ij}. The update rule is characterized as follows for a certain undirected weighted graph $G = (V,E,w)$:

$$s_i(t+1) = \frac{w_{ii}s_i(t) + \sum_{j\in N(i)} w_{ij}s_j(t)}{w_{ii} + \sum_{j\in N(i)} w_{ij}} \text{[48]} (Eq.7)$$

3.2.4 Friedkin-Johnson Model

An extension that outperforms the DeGroot model is the Friedkin-Johnson (FJ) model. The approach suggests that people have two opinions: internal and expressed. Internal views are said to be innate and represent a person's unique point of view. This is the viewpoint that a person may hold without outside influences. However, in reality, friends or neighbors may affect a person's proclaimed attitude due to factors like the need for social acceptance. The FJ model models this impact by weighing the average of a person's own view and a neighbor's declared opinion. These opinions are developed by iterative averaging. The FJ model's stated opinion vector is the Nash equilibrium of an opinion-forming social game in which players are rewarded with social costs[49].

Let the network $G = (V,E,w)$ consist of the set of edges $E \in V \times V$, the strip edges $m = |E|$, the weight function w of the edges E mapping to their nonnegativenonnegative weights $W(E)$, and the set of nodes $V = \{1,...,n\}$ that represent the set of nodes. To represent the network, where when $\{i, j\} \in E$, $W_{ij} = W(i, j)$, and otherwise$W_{ij} = 0$, we use a weighted adjacency matrix (with zero diagonal) W. $N(i)$ indicates the group of nodes that are close to a node, $N(i) \triangleq \{j \in V \mid (j,i) \in E\}$, or the friends nodes that have an impact on the node i in the social network. Let e be a vector with the appropriate size. Let and indicate the diagonal matrix $D \triangleq diag(d)$ and the vector $d \triangleq W^T e$ that contains the weighted incidence of all nodes, respectively. Explain what a Laplace matrix is $L \triangleq D - W$. In this case, the symbols pertain to the directed network's in-degree, equivalent to the undirected network's degree (either in-degree or out-degree).

The FJ model differs from the Degroot model in two ways: each person has a distinct personal opinion s_i and is free to voice in public in z_i. The opinion of each node is essentially a weighted average of the opinions of its neighbors and itself.

$$z_i = \frac{w_{ii} s_i + \sum_{j \in N(i)} w_{ij} z_j}{w_{ii} + \sum_{j \in N(i)} w_{ij}} (Eq.8)$$

4 Improved models based on classical models

4.1 Discrete models

4.1.1 Voter model

In physics, where researchers often concentrate on homogenous spatial structures and translation invariants, such as the regular lattice, the voter model and its variations are of tremendous interest to study. However, in social physics, the regular lattice only offers an approximate approximation of geographic closeness. Because of this, researchers have started to model voters on intricate social networks, albeit few recent studies have considered the dynamism of actual social networks, which might affect how people interact. For instance, whether or not two individuals have a mutual friendship depends on whether they hold the same opinion. Opinion dynamics, therefore, operate on networks that change adaptively. Additionally, the network's topology may affect how views evolve; as a result, the dynamics of an adaptive network encompass both the evolution of the network's topology and the development of opinions. Due to the connection of these two processes, nodes and links can grow concurrently, a phenomenon that illustrates how people's beliefs influence how they link and how their new ties are formed. Understanding the evolution of opinion in dynamic networks is crucial[50].

4.1.2 Majority Adjudication Model

Most decision models can filter the content that social network members are exposed to, enhancing marketing efforts and reducing bias in information filtering. It is simpler for people to be exposed to material that supports their ideas, thanks to the creation of filter bubbles. Based on this background, researchers like A.L.M. Vilela, L.F.C. Pereira, L. Dias, and others have refined the majority adjudication model. By adding a visibility parameter V , which has a value reflecting the likelihood that a person overlooks their neighbors' opinions, they dynamically updated the majority adjudication model. For instance, $V = 0.5$, which means that, on average, each person at that moment disregards the opinions of half of the nearby nodes. The visibility parameter$V$within the range of $0 \le V \le 1$ indicates the possibility that a particular person will impact the consensus viewpoint when one of their neighbors' opinions is considered. The social interaction structure supports both ordered and unordered phase transitions when the social temperature rises over a certain level. The visibility parameter has a positive correlation with this crucial value.

In social networks, a person's closest social circle frequently affects them most. Therefore, managing a person's visible neighbors may enhance social networks and lessen polarization. We may successfully encourage the peaceful growth of social networks by modifying a person's nearest contact group[51].

4.1.3 Sznajd Model

Sznajd, the model's original proponent, considered followership and independence as two distinct kinds of social influence. Where p represents the chance of independence and 1-p is the likelihood of a follower. Additionally, K. Sznajd-Weron, M. Tabiszewski, and others suggested modifying Sznajd's model in which people exhibit follower behavior by adhering to group standards. The voter model or the majority model may also be used to simulate this kind of social impact. Social conventions do not constrain individuals who behave autonomously and are free to alter their minds or actions at any time. It is crucial to remember that even while acting independently, a person can still alter their mind; however, this shift is independent of societal conventions. Tabiszewski created a flexibility factor to more accurately estimate the amount of conservatism in society, which measures the likelihood of altering one's mind in the presence of independent conduct[52].

4.2 Continuous Models

4.2.1 Deffaunt model

Zhang Li and Liu Yun provide a unique model of opinion development based on how people behave when they communicate with others about their points of view in real life. With the help of this paradigm, people may autonomously alter their perspective engagement techniques to suit various demands and circumstances. This adaptive process significantly improves the adaptability and flexibility of opinion evolution. The model also considers the memory of those forming opinions, which is crucial in reproducing the process of opinion evolution in real societies.

To enable the interacting parties to have diverse opinion interaction parameters, the study first enhances the individual opinion interaction parameters in Deffaunt's model. We may use x and x' to represent the opinions of two randomly chosen people in the original Deffaunt model. We limit the perspective engagement d to ensure two people can only communicate when $|x-x'|<d$. The following two new guidelines can be used for opinion interactions:

$x = x + \mu(x' - x)(Eq.9)$

$x' = x' + \mu(x - x')(Eq.10)$

The Deffaunt model heavily relies on parameters μ. μ may be changed to produce groups with various characteristics. For instance, when $\mu = 0$, the persons participating in the dialogue will not alter at that point. When $\mu = \frac{1}{2}$, the participants in the interaction will receive the mean value of the two points of view. These two examples show interacting people with various characteristics. People are more tenacious and resistant to changing their opinions when μ is little, and they are more inclined to abandon their plans when μ is larger. Regarding the evolution of public opinion in a real society, the fixed constraint of the parameter μ in the original Deffaunt model on the interaction of people is illogical. For people to freely select the parameter μ, the memory attribute of persons is added as part of the model improvement process.

Additionally, the individual can modify their interaction strategy by the interaction strategy selected by the other party by adding the "memory" feature to the individual. These changes can increase the effectiveness and synergy of interpersonal engagement. To further improve the person's effectiveness in social interactions, autonomous selection of the parameter μ can be accomplished by considering the person's memory.

The concept includes a group of people referred to as extreme persons who are unrestricted in social interactions and have strong convictions in their opinions. Though extreme people also adhere to a set of opinion interactions, the extent of their perspective modifications is minimal.

The revised model expands the Deffaunt model by including individual memorability and variable strategy choice. It also offers a new method for the study of opinion evolution. At the same time, the enhanced model is more based on the traits of individual interaction and opinion evolution in actual society, which aids in understanding how genuine opinion events emerge[53].

4.2.2 HK model

Other better versions exist based on the HK model. To examine the impact of group pressure on the viewpoint evolution process in cooperative and cooperative adversarial networks, Zhang Shanqi suggested two models based on bounded trust viewpoint dynamics.

According to this approach, changing a person i's perspective involves two stages. To create their internal views x_i, each person talks with others inside their trust boundary in the first stage, much like the old HK model. The individual i's viewpoint is impacted by group pressure x_{avg} in the second stage, and both the individual's internal viewpoint x_i and the group's stated opinion contribute to the individual's expressed viewpoint. The mathematical model below may be used to explain this process:

$x_i(k+1) = (1-a)x_i(k) + x_{avg}(k)(Eq.11)$

The inner opinion attained when a person i talks with those inside his or her trust boundary is indicated as $x_i(k) = \frac{\sum_{j\in N_i(k)} w_j x_j(k)}{\sum_{j\in N_i(k)} w_j} \in [0,1](Eq.12)$. In contrast, the k manifestation of the group viewpoint is designated as $x_{avg}(k) = \frac{\sum_{j\in V} w_j x_j(k)}{\sum_{j\in V} w_j} \in [0,1](Eq.13)$. While the group pressure coefficient $a \in [0,1]$ depicts the degree of effect of the group perspective on the individual's stated opinion, the weight coefficient of an individual $w_j > 0$ depicts the degree of influence of individual i expressed position on other people[54].

4.2.3 Degroot model

The DeGroot model has undergone extensive development thanks to the efforts of various scholars. The literature [55] presented the FredkinJohnson (FJ) model, expanding the DeGroot model after evaluating the impact of various levels of individual stubbornness on the original viewpoint. In contrast, literature [56] uses the DeGroot model and social power theory to examine how a person's social power changes throughout a series of questions. Additionally, Qinyue Zhou and Zhibin Wu created an opinion development model based on the DeGroot model that considers the variations in the decision-maker's roles in literature [57].

4.2.4 FJ model

The FJ model is an enhanced version of the Degroot model, as was discussed in the preceding section. However, some researchers have modified the FJ model even further to provide a more concentrated and enhanced FJ model. For instance, Anton V. Proskurnikov considered crucial stability and convergence speed aspects of the FJ model of perspective dynamics. Additionally, he examined the FJ model's extension in the presence of time-varying social forces and established the necessary criteria for its stability[58].

5 Discussion

This paper introduces the current research status in opinion dynamics based on representing opinions from two dimensions: discrete models and continuous models. First, the paper introduces the discrete models, including the Voter model, Majority Adjudication Model, and Sznajd Model, where each social network member only has access to two alternative points of view and goes through the use cases and methods for each model. Then, continuous models, including the DW, HK, FJ, and Degroot models, are discussed, and the related research work is based on these models. This paper also presents the results of improved models for the discrete and continuous models, respectively.

Although there has been significant advancement in the study of opinion dynamics, many models still have not achieved their theoretical potential from the perspective of the control group. When seen from a sociological perspective, the present mathematical models make relatively basic assumptions and fail to consider how various social aspects interact during the modeling process. In-depth research should be done in particular on the following topics:

(1)Many open issues, such as the effect of network structure, remain unresolved in the existing models. Each person in the Voter model is linked to a few neighbors, so they may change their opinions by talking to their neighbors. As a result, the network structure has a crucial impact on the voter model since it establishes the neighbor relationship between individuals, i.e., which persons may influence one another. Additionally, different network topologies may produce various outcomes about the spread of ideas. Similarly, the Majority Adjudication model builds individual connections in how opinions spread. In this concept, each person chooses his or her opinions by secretly voting with his or her neighbors, and the result is likely to reflect the majority view of those neighbors. Both the Voter model and the Majority Adjudication model depend to some extent on the network structure since they both rely on the neighbor relationship for opinion dissemination. Therefore, future opinion dynamics research can concentrate on studying the impact of network structure. Majority opinions and convergence rates may vary depending on the network topology.

(2) Without considering the issue of individual disparity, there is no universal model that can represent group behavior in social networks. For instance, the DeGroot model overlooks the variety and individual distinctions among people, presuming that there are no differences among people and that people simply adopt beliefs through conversation with their neighbors. In reality, people differ and are diverse; for instance, they all come from various origins and hold different opinions and values. The implications of individual variations on the process of opinion construction and evolution and changes in individual weights or opinion update criteria have not yet been thoroughly researched.

6 Conclusion

This study initially explains the function and significance of opinion dynamics and information related to graph theory and social networks as basic knowledge foundations. This study also examines and assesses current research on two categories of models, discrete and continuous. Then, it provides a forecast for the advancement of opinion dynamics in the future. To better understand and interpret the quality and reliability of the existing studies, this paper integrates and analyzes the results of the literature on opinion dynamics that has already been published. It also offers insights into the relationships and patterns among various opinion dynamics models for future research.

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