The Application of Derivatives in Enhancing Economic Decision-Making

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Abstract:

This paper explores the specific application of derivatives in economic analysis, aiming to provide an effective mathematical tool for economic analysis to improve decision-making efficiency and accuracy. Through two specific cases - the profit maximization problem of fast-moving consumer goods companies and the profit maximization problem of retail enterprises, it can be demonstrated how to use derivatives to solve and analyze economic models, and then propose corresponding optimization suggestions. In addition, through detailed mathematical derivation and data analysis, how derivatives can help enterprises optimize decision-making and improve economic efficiency was analyzed. Research has shown that derivatives, as an important tool in economic analysis, can effectively enhance the scientifically and accuracy of decision-making.

Keywords: Derivatives, Profit maximization, Economic analysis

1. Introduction

The definition and importance of economic analysis: Economic analysis refers to the use of economic theory, mathematical models, and data analysis methods to study economic phenomena and their laws, in order to provide decision-making basis and policy recommendations. Its importance lies in helping decision-makers understand market mechanisms, predict economic trends, optimize resource allocation, and formulate effective economic policies.

The basic concept of derivatives in mathematics: Derivatives are tools that describe the rate of change of a function at a certain point. In mathematics, if the derivative (f '(x)) of a function (f (x)) at point (x) exists, it represents the instantaneous rate of change of the function (f (x)) with respect to (x). The calculation of derivatives is based on the concept of limits and is one of the foundations of calculus. In addition, derivative is a description of the speed of change in the dependent variable caused by changes in the independent variable. In economic functions, derivative is an analysis of absolute increment and absolute rate of change. However, in real life, demand changes caused by price fluctuations are often encountered, and relative increment and relative rate of change need to be applied for analysis of changes. This analysis method is called elasticity analysis [1].

The application and role of derivatives in economic analysis: Derivatives are mainly used in economic analysis to study marginal effects, optimization problems, and dynamic changes. Through derivatives, economists can analyze the marginal cost and marginal benefit of a product, maximize profit or utility, and study the rate of change in market demand. In economics, the application of derivatives helps to understand and solve practical economic problems such as firm pricing, production decisions, and market forecasting.

Common application scenarios of derivatives in economics:

1. Marginal analysis: The calculation of marginal costs and marginal benefits is a typical application of derivatives, used to determine production and pricing strategies.

2. Optimization problem: Using derivatives to solve problems such as maximizing profits and minimizing costs, helping companies formulate optimal strategies.

3. Market forecasting: Derivatives are used to analyze changes in demand and supply functions to predict market trends and price changes.

Research purpose and significance: This article aims to explore the specific application of derivatives in economic analysis and demonstrate how derivatives can play a role in practical problems through case analysis. The significance of research lies in providing powerful mathematical tools for economic analysis, improving decision-making efficiency and accuracy, and enabling decision-makers to optimize resource allocation and formulate policies more

scientifically.

2. Methodology

2.1 Research Method

Based on a case study approach, this research demonstrates the application of derivatives in economic analysis. The main steps of the analysis include: first, establishing economic models based on real-world problems; second, expressing these economic models through mathematical expressions; third, using derivatives to solve the models and identify optimal solutions; and finally, analyzing the results and offering recommendations for optimization.

2.2 Data Sources

The data for the case analysis comes from publicly available corporate financial statements, market survey reports, and related literature.

3. Results

3.1 Case 1: Profit Maximization Problem

Background
A fast-moving consumer goods company seeks to find the optimal pricing strategy for its products to maximize profits. Suppose the company's product demand function is in the equation (1) to equation (12)

$$Q = 1000 - 20P \tag{1}$$

$$C(Q) = 1000 + 10Q \tag{2}$$

Problem Description
The company needs to determine the optimal price (P) to maximize profit (Pi). The profit expression is Pi = R - C, where (R) is revenue and (C) is cost.
According to the demand function, revenue (R) can be expressed as:

$$R = PQ = P(1000 - 20P) \tag{3}$$

Cost (C) is:

$$C = 1000 + 10(1000 - 20P) \tag{4}$$

Derivative Solution
To find the profit-maximizing price, we first derive the profit expression:

$$Pi = P(1000 - 20P) - [1000 + 10(1000 - 20P)] \tag{5}$$

Simplify the above expression:

$$Pi = 1000P - 20P^2 - 1000 - 10 times 1000 + 200P = 1000P - 20P^2 - 11000 + 200P = -20P^2 + 1200P - C11000 \tag{6}$$

Take the derivative:

$$dPi / dP = -40P + 1200 \quad Set dPi / dP = 0 : -40P + 1200 = 0 \tag{7}$$

$$Solve for (P) : P = 30 \tag{8}$$

Thus, the optimal pricing for the product is 30 yuan. For any product in the market, there are total input costs and total revenue, and the total profit function represents the difference between total revenue and total cost, that is, the remaining part after deducting costs from revenue, using total revenue minus total cost. If L is used to represent the profit function, according to the income function (rig) and cost function C (g), it can be known that L=R (g) - C (g) [2].

3.2 Case 2: Revenue Maximization Problem

Background
A retail enterprise aims to find the optimal advertising expenditure to maximize revenue. Suppose the enterprise's revenue function is:

$$R(A) = 500 + 50A - 2A \tag{9}$$

where (A) represents advertising expenditure.
Problem Description
The enterprise needs to determine the advertising expenditure (A) to maximize revenue (R). The revenue expression is:

$$R = 500 + 50A - 2A^2 \tag{10}$$

Derivative Solution
To find the revenue-maximizing advertising expenditure, we first take the derivative:

$$dR / dA = 50 - 4A \tag{11}$$

$$Set dR / dA = 0 : 50 - 4A = 0$$

$$Solve for (A) : A = 12.5 \tag{12}$$

Thus, the optimal advertising expenditure is 12.5 million yuan. Profit is equal to revenue minus cost, and marginal profit is the difference between marginal revenue and marginal cost, i.e. MI=MR-MG. When MR-MC>0, for every additional unit of product, the increase in revenue is greater than the increase in cost, resulting in an increase in total profit. However, it has not reached the scale to obtain maximum revenue. At this point, the enterprise should expand its production scale. When MR-MG<0, for every additional unit of product, the increase in revenue is less than the increase in cost, resulting in a decrease in total profit, indicating that the enterprise should reduce production scale [2-4].

4. Literature References

In economic analysis, derivatives, as a key mathematical tool, have been widely used in various economic models and decision analysis. The following review provides a brief overview of the main viewpoints, research methods, and conclusions of relevant literature, highlighting the connections and differences between the literature. The development of the derivatives market has long been a topic of interest among researchers, policy makers, and fi-

nancial agencies. It directly plays a vital role in a financial system and greatly contributes to various aspects of an economy as a whole [4-6].

Firstly, the Business and Economics Research Group of Ho Chi Minh City Open University explored the role of derivatives in economic optimization in their research "Derivative Application in Economic Optimization". This study adopted mathematical optimization and marginal analysis methods, focusing on how to use derivatives to determine the optimal level of production and cost. Research has shown that derivatives are crucial in determining marginal costs and marginal benefits, helping businesses maximize profits in competitive markets.

Related to this, the research team of Banking University Ho Chi Minh explored the application of derivatives in market equilibrium analysis in "Using Derivatives for Market Equilibrium Analysis". This study used dynamic models and numerical simulation methods to analyze the impact of changes in prices and demand on market equilibrium. Research has shown that derivatives can help economists predict market changes and adjust price strategies to achieve market equilibrium.

In addition, another article titled "The Application of Derivatives in Economic Analysis" published by Yang Xiong from the Public Course Teaching Department of Loudi Vocational and Technical College and Zhou Lifen from the Mathematics Group of Shaoyang County No.1 Senior High School elaborates on the direct application of the derivative definition reflected in the calculation of demand elasticity and the limit thinking of increment ratio. Research has shown that there is no fundamental difference between point elasticity and arc elasticity, with point elasticity being the ultimate form of arc elasticity.

Overall, although these literatures have explored the application of derivatives in economic analysis, their respective research focuses are different. Ho Chi Minh City Open University's research focuses on production and cost optimization, while Banking University's research focuses on market equilibrium, and other research focuses on consumer behavior analysis. The commonality among the three is that they all demonstrate the important role of derivatives in economic analysis, while also reflecting the diversity and applicability of derivative tools in different economic problems [6-10].

5. Recommendations

Based on the analysis, the following suggestions are proposed:

Enterprises should strengthen the application of mathematical tools, especially derivative analysis, to optimize decision-making. In practice, combining market research

data with internal company data to establish and solve more accurate models. Regularly conduct economic analysis and adjust business strategies in a timely manner to respond to market changes.

5.1 Using Derivatives for Prediction and Decision-making

Derivatives can not only help businesses optimize existing strategies, but also be used to predict market changes and make future decisions. By establishing relevant economic models and conducting derivative analysis, enterprises can anticipate market trends and potential economic benefits, thereby making more scientific decisions.

5.2 Implementation Steps and Expected Results

Implementation steps: Enterprises should first conduct detailed data collection and model establishment, then apply derivatives for analysis, and finally adjust strategies and carry out practical operations based on the results.

Expected effect: Through effective derivative analysis, enterprises will be able to improve the scientifically and accuracy of decision-making, thereby gaining competitive advantages and economic benefits in the market. Optimization analysis is one of the core issues in economic management activities, usually using mathematical analysis tools such as derivatives and linear programming to explore the behavior that maximizes or minimizes the cost of economic activities, to provide relevant decision-making basis for enterprise managers. The main optimization problems in economic analysis include output maximization analysis, income maximization analysis, profit maximization analysis, optimization analysis of rational resource utilization, cost minimization analysis, and optimal combination analysis, which are usually accompanied by some constraints. Optimization analysis can help enterprise managers seek to maximize enterprise profits and minimize production costs and management expenses, and its significance is very profound.

6. Conclusion

Through the above case analysis, we can see that derivative play an important role in economic analysis. It can not only help companies find the optimal pricing and advertising investment strategy, but also improve the scientific and accurate economic decision-making. Specifically, through derivative analysis, the relationship between different variables can be quantified to find the conditions that maximize economic benefits. This article verifies the effectiveness of derivatives in practical economic decision-making through specific case analysis. The process of solving the problems of maximizing profits and max-

imizing profits demonstrates the role of derivatives in optimizing decisions and provides specific optimization strategies for enterprises.

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