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Analysis of the Application of Game Theory from Multiple Aspects: Economics, Business, and Education

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Abstract:

In the late 20th century and early 21st century, there has been extensive literature published regarding proposals of the utilization of game theory from different academic aspects, which are tedious and unnecessary work for scholars who only wish to simply learn about some of the applications of game theory. This paper summarizes and organizes a few examples of the applications of game theory using prominent existing literature from three aspects: business, economics, and education, hoping to spark scholars' interests and provide a broad understanding for readers as to when and how game theory may be used. When applying game theory to a real-world issue in business, economics, or education, the development and design of a "game" with certain selected constraints and variables is used to replicate the real-world model using mathematical languages. Depending on the real-world conflict these models are portraying, they might be novel concepts or variations of already-existing models. The summarization of some applications of game theory may spark interest for scholars interested in the various fields explored such as business management, economics, and optimization and quickly understand current progress in the area of game theory.

Keywords: Game theory; Cournot's duopoly model; Prisoner's dilemma; equilibrium refinement.

1. Introduction

From childhood memories, rock-paper-scissors, to financial investments as grown-up and skillful adults, human lives have always been intimately linked with a ubiquitous concept known as game theory. Game theory is a prediction and optimization of the profit of the individuals as they are encountered with another individual with a conflicting interest using analysis and mathematical models. The presence of a game theory is inevitable when interactions between different individuals of independent thinking occur [1, 2]. Although some fundamental ideas regarding concepts of game theory appeared as early as 0-400 common era ago, it was really modern works in the nineteenth and twentieth century that set the foundation for the analysis and interpretation of strategic games [1]. Between 1950 and 1953, Nash proved the existence of the Nash Equilibrium, which then became the central idea of game theory and extensively used in economics and optimization [3]. In the late 20th century, the concept of game theory began to show appearance in different academic fields such as economics, business, and education, with its increasing popularity and publications.

The study of economics is heavily based upon the belief of methodological individualism, stating that all social phenomena are closely associated with individual behaviors and thereby forming predictions and optimization strategies through analyzing the individual behaviors and environmental factors [4]. The study of business comes inevitably with the intercommunication between suppliers, coworkers, employees, organization, customers, and competitors which come intimately with every decision and negotiation made by any business or organization [5]. Similarly, the study of education is all about relationships between organizations, professors, students, and guardians [6]. These fields of academic study that appear to be parallel lines are all heavily dependent on human interactions with others and their environments which parallelly prove the potential application of game theory in seeking an "optimal solution".

In 2016, Samuelson analyzed the application and presence of game theory in economics through stating the utilization of the Nash Equilibrium and Cournot's model of imperfect competition by economists [4]. Kurz researched specifically upon the application of game theory in public economics, such as the allocation of local public goods associated with the local community and distribution of wealth and income [7]. In August 2003, Erhun and Keskinocak presented the business application of game theory on price and quantity competition, supply chain coordina-

tion, and games of complete and incomplete information [5]. In 2015, Wei proposed the potential utilization of game theory particularly in business management using classical game theory models such as the prisoner's dilemma and the clever pig game [8]. In May 2009, Law and Pan presented a game theory analysis of the private education legislation in China focusing the use of game theory during negotiations with the national department of education to amend existing regulations or form new innovative regulations [9]. In 2016, Beltadze generalized the study of Law and Pan exploring the noncooperative game between the professors and students as a game theory that could be solved using the Nash Equilibrium [6].

Although having many literatures focusing on the application of game theory from one specific aspect such as the literatures explored above, these literatures are not appealing to individuals seeking a general understanding of game theory. Minimal work exists in the general analysis of the application of game theory, except for certain books that may be too tedious and lengthy for scholars who are simply interested. Through understanding and inspecting prominent existing work of the application of game theory in different academic field, this paper aims for organizing published work of game theory by summarizing and reinstating key concepts of the application of game theory from three main aspects: business, economics, and education, each intertwined with interactions and communications between interests contradicting individuals and coalitions. In addition, this paper presents some disadvantages of game theory specifically emphasizing the limitations of Nash Equilibrium and prerequisite for the application of game theory [2].

2. Game Theory

2.1 Definition of Game Theory

Game theory is a prediction and optimization of the profit of the individuals as they are encountered with another individual with a conflicting interest using analysis and mathematical models [1]. Game theory analyzes daily human interactions believing that each individual adheres to a specific set of strategies and is aware of their action's impact to the outcome [2]. Therefore, a game will consist of three elements, a set of players, a set of actions for each player, and a set of preferences, otherwise known to be the utility function [1]. Amongst the broad topic of game theory, the games are categorized into different groups depending on the special relationship between the three elements. For instance, when the sum of the gain of each player is equal to zero, a game is said to be a zero-sum game, where a "win-win" scenario is not possible [10, 11]. The maximization of a certain player's expected gain without harming the other player's benefit thereby became the fundamental idea of game theory. This optimization can be found using the Nash equilibrium introduced above [10].

2.2 Nash Equilibrium

Nash Equilibrium is a balance point where an individual's strategy is the most ideal response against the Nash equilibrium strategies of the other individual. Given that s_i^* is the Nash equilibrium strategy for player i and s_{-i}^* is the Nash equilibrium strategy for all player excluding player i, then player i's gain from playing s_i^* with their opponent playing s_{-i}^* is greater than player i's gain from playing any other available strategy [10].

$$u_i\left(s_i^*, ? s_{-i}^*\right) ? \geq u_i\left(s_i ?, ? s_{-i}^*\right) ? \quad (1)$$

Furthermore, a game may have a pure strategy Nash equilibrium or a mixed strategy Nash equilibrium, depending on the point of balance as a game g is played multiple times. A pure strategy Nash equilibrium indicates the consistency of the point of equilibrium or best strategy, suggesting the unnecessity of playing a certain game repeatedly. Oppositely, the mixed strategy Nash equilibrium is used to model games where the best strategy for each player is not fixed for each time the game is played repeatedly, often due to the role of probability [12].

3. Game Theory in Economics

3.1 Selection of Variables and Constraints

In early years, game theory was based upon a classical view where all variables, prerequisite, and potential deviations from a certain strategy are all inclusive in a model, rather than providing just an approximation. The idea that it is sometimes impossible, unnecessary, or nonoptimal to model a realistic economic situation complicates the study of game theory in economics. For instance, Cournot's duopoly models the choice of a firm's output quantity which is then sold at a common market price, providing the total produced quantity in the market. In 1883, Bertrand claimed and published a variation of Cournot's model where the firms are choosing prices instead of the quantities of output. The difference is, Cournot's model leads to positive profits for the firm while Bertrand's model is forced to lead to zero profits for the firms. In the classical view described above, simply observe the general actions of firms in the real world, and decide which model to utilize in modeling that particular situation. This example is one of many in which there are complex considerations from multiple aspects regarding the inclusion and exclusion of variables and constraints depending on

the objective result [4].

3.2 Multiplicity of Equilibria

Realistic models of game theory with overwhelming numbers of variables may often result in a set of equilibrium, sparking the exploration of equilibrium refinements amongst scholars. The foundational idea of equilibrium refinement is the formation of a subset of the set of Nash equilibria by setting upper and lower bounds using rationality. For instance, in the example above, an optimal solution should not include a production quantity more than the monopoly quantity regardless of its opponent's decision. By repeating the process of minimizing the range and thereby the number of Nash equilibrium outputs, one may reach one rational strategy which is the resulting Nash equilibrium output. Nevertheless, certain cases using probability and randomization may not be able to be evaluated using rationalizability [4].

3.3 An Example with Public Economic

Given the utilization of game theory in economics above, one may form a simple allocation of pure public good scenarios by introducing variables and constraints and making three assumptions. First, there should be no exclusion of consumers, meaning that any individual has the ability and access to all available supplies. Next, each consumer has the ability and power to not use any available supply depending on their need and interest. Finally, the number of consumers should not exceed the set amount, causing congestion. This identifies the complexity in modeling economies with public goods, as these strict assumptions are not always fulfilled. For instance, local public goods violate the first assumption where consumers away from a town may not enjoy the local public good of that specific town [7].

4. Game Theory in Business

4.1 Prisoner's Dilemma in Business

The classical game theory model of the prisoner's dilemma is extensively used in business to model many realistic scenarios. The prisoner's dilemma is a game between two criminals that was arrested by the police and brought to two separate rooms to be interrogated (Table 1 shows the example). Each criminal has two choices, or strategy, stay silent or testify, each resulting in different years in prison for both players such as the payoff table below [13].

In a prisoner's dilemma, the best choice for both criminals is to stay silent. However, the idea that testifying becomes tempting as there is a higher gain for the criminal if the other criminal stays silent as promised. This tests that both criminals staying silent is not a Nash equilibrium and forms an alliance that is not self-enforced. Oppositely, both criminals testifying is a Nash equilibrium, automatically enforcing an alliance that is not easily broken. For example, in 2012, nine mobile-phone companies met in a price union meeting, requesting that all members shall not reduce the price, raising the profit of each company. Nonetheless, the members did not truly believe that their opponents would adhere to their agreement, so one started to reduce the price which resulted in the bankruptcy of the phone price alliance. In this example, the agreement of all members to not reduce their price was not bonded and enforced with the violation of the member's profit if one chose to break the agreement, thus having a high risk of one member deviating from their agreement [8].

Table 1. Payoff Table for the Prisoner's Dilemma Model

Player A \ Player B	Stay Silent	Testify
Stay Silent	(1, 1)	(15, 0)
Testify	(0, 15)	(6, 6)

4.2 Business beyond Prisoner's Dilemma

Most of the time, business scenarios go beyond a simple model of the prisoner's dilemma, a complete information static game. A "game" in business is often played multiple times, changing the best strategy that a company may choose. To give an example, a prisoner's dilemma between two businesses could result in a stay silent-stay silent strategy as this decision is bonded with the agreement that this same "game" would be played numerous times in the future and that being pleasant in this "game" would increase the change of cooperation in the days to come. Therefore, many business "games" may not be a complete information game nor a static game [5].

5. Game Theory in Education

5.1 Game Theory in Higher Education

By viewing the interaction between the professors and students as a noncooperative game with S organizational system of P professors and K students, the optimal solution could be represented using the democratic model,

emphasizing the mutual responsibility and participation of the professors and students in a learning process. Although a game theory representation may not compose of all potential variables and cases, it forms a general framework for understanding and improving the dynamics between students and educators, ensuring that decisions benefit both parties and lead to effective education [6].

5.2 Game Theory in Education Legislation

There have been many previous attempts to model China's law-making politics using four major models: the command model, leadership struggle model, organizational politics model, and garbage can model. However, Law and Pan argue its inability to accurately explain the complex dynamics between China's legislative and administrative bodies, particularly in legislating private education. They introduced the inventive use of game theory and bounded rationality as framework to provide insights on the negotiation, cooperation, and competition among China's lawmaking bodies [9].

5.3 Limitation of Game Theory

Though being a mathematical theory of optimal strategy, there exist multiple limitations, given by the definition of game theory, in which game theory is not applicable. For instance, the use of game theory requires all players to have access to equal amounts of information and where all players are rational and intelligent [14]. Game theory uses real values to evaluate an optimal solution, however, realistic cases may sometimes be hard to reach an accurate numericized value to be used in a game theory. Also note that the application of game theory in economics, business, and education described above are all approximations and references for actual decision making.

6. Conclusion

Game theory has a wide range of applications given its flexibility and freedom in modeling various scenarios requiring optimization. In business, economics, and education, the utilization of game theory to proceed a real-world conflict requires the creation and design of a "game" with constraints and variables to best recreate the real-world model using mathematical languages. These models may either be a variation of an existing model or a completely new concept depending on the real-world conflict it is modeling. Through the explanation of a few examples of the application of game theory in business, economics, and education, this paper made its main contribution in providing some ideas and concepts of game theory's application for scholars interested in the field to better understand and be introduced to a few of the current progress in game theory. The study of the application of game theory allows scholars to visualize concepts of game theory such as the Nash equilibrium and classic models such as the prisoner's dilemma using specific examples. However, many concepts of game theory still remain relatively unexplored, affecting the popularity of game theory in some academic fields. For instance, the equilibrium refinement, in game theory explained above in 3.2 cannot be used to refine all problems that come with multiple Nash equilibrium and will only give one solution in certain cases. Future investigations could focus on the exploration of the concept of a universal equilibrium refinement that may consist of a set of strategies, depending on the characteristic of a certain model, that may be used for all models.

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