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Analyzing the Characteristic of Simplex Algorithm

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Abstract:

Simplex algorithm is a kind of iterated algorithm which is used to solve linear programming problems. It is raised by George Dantzig in 1974 and is still a valid tool especially when the scale of problem is relatively small or secondary. Simplex algorithm starts at an initializing feasible solution, and constantly approach to the optimal solution until the optimal solution is found or unbounded. Serving as a classical algorithm solving linear programming problem, it plays an important role in both theoretical. With the continuous development of computing technology, the simplex method and its varieties will continue in optimization. This research aims to help people make better decisions in optimization problems by summarizing the formula, characteristics and problem solutions of simplex algorithm. The summary is come from existing literature and including the condition of using simplex algorithm, all kind of different solutions, an example of simplex tableau method and the future of simplex algorithm.

Keywords: Simplex algorithm; linear programming problem; summary.

1. Introduction

Linear programming is an important component of optimizing realistic and academic problems [1]. It not only makes an achievement in practice, but also gains theory evidence in mathematics [2]. It is widely used in fields like economic management, engineering technology, military operations, transport services and even in environmental protection [3, 4]. An important mission for linear programming is to serve for modern production and management [2]. One of the most frequently used method for solving linear programs is simplex algorithm, which was firstly invented by Dantzig [5]. Klee and Minty used a problem in linear programming which shows the worst-case running time for the simplex algorithm using Dantzig's rule in 1972 [6]. In 1984, Karmarkar proposed a new polynomial-time algorithm called the interior point method for a linear programming problem to respond to this [6].

In the following decades, simplex algorithm was developed quickly, which helped accelerate the progress in optimization problems. Many contributions have been made in optimization problems, which makes research more convenient and easier to be carried out. In 2013, Gopalakrishnan developed the Particle Swarm Optimization [7]. In 2014, Ghaemi and Feizi-Derakhshi developed the Forest Optimization Algorithm. In 2022, Chopra and Mohsin developed the Golden Jackal Optimization [8].

The algorithm is also optimized to improve its accuracy and solving efficiency in some specified problems. The intuitive and understandable method is now being calculating improvement. Researchers are trying to make simplex algorithm more efficient and more stable. This method is also trying to be applied to non-linear programming problems and is still a simple and convenient method to be used. Many investigators are devoting themselves to simplify this method when analyzing massive calculations. It will also contribute to game theory like zero-sum games, potential games and zero-sum potential games.

Simplex algorithm has many advantages. Firstly, it has strong universal property. It can process different kinds of linear programming problems without special conversion and pretreatment of the problems. In addition, the algorithm of this method is relatively steady. It aims to get the best solution, so nearly every step of the process is feasible. Thirdly, this method is easy to implement. Though the theory of proof is relatively hard to complete, users can solve their problems through coding intuitionistic steps into programming language. Moreover, simplex algorithm has a long historical standing and is fully validated. It has experienced decades of development and perfection, so it has convincing reputation in industrial and academic circle. What also worth noticing is simplex algorithm can help improve the health level of human. It can help decide the usage and dosage of medicine and the optimal surgical methods, which can maximize the effectiveness of healthcare delivery.

Being a classical method of solving linear programming, simplex algorithm can be used to solve problems in economy, administration, engineering, scientific experiments and even in biochemistry [9, 10]. It provides a valid tool to help calculate the resource allocation, production designing, route planning, cost minimization and profit maximization. In pace with the improvement of computer science, the applied range and capability of simplex algorithm is expanding. The researching of simplex algorithm improves the integration of numerical calculation, theory optimization and artificial intelligence in computer science. Meanwhile, the majorization of simplex algorithm also needs the support by modern technology. In the aspect of researchers, discussing simplex algorithm can foster the scientific and creative thinking skills, which can improve their overall qualities and competitive power.

Simplex algorithm has some researching blank, though. Firstly, it is not widely used in integrating with other algorithm, so discussing how to improve this method into mixed algorithm is an important research area. In addition, when directing into novel and realistic problems, how to expand simplex algorithm to suit new application environment is a constant research subject.

To sum up, this project summarizes and analyzes the features and formula of simplex algorithm. It aims to provide the decision makers for more scientific and reasonable solutions when making decisions, and popularize this method to simplify more problems in daily life and in academic fields.

2. Preliminaries

2.1 Convex Polyhedral

The condition of calculation linear programming problems by simplex algorithm is: the equation should be convex. The definition of convexity is:

For every two points $x, y \in X$ also contains the segment xy, the set $x \subseteq \mathbb{R}^n$ is convex [11]. If for every $x, y \in X$ and every $t \in [0,1]$, the function $f : X \to \mathbb{R}$ is called convex, so:

$$f\left(tx + (1-t)\,y\right) \leqslant tf\left(x\right) + (1-t)\,f\left(y\right) \tag{1}$$

If the inequality is strict for all $x \neq y$, the function is called strictly convex [11].

2.2 Conditions

The constraint conditions of the maximize or minimize problem should be linear, and the constraints can be equations or inequations. The problem must have at least one feasible solution, and suitable calculation resources (if the process is massive). When dealing with maximizing problems, multiply the equations with -1 will be a convenient method to transform them into minimization problems.

2.3 The calculations between matrices

Matrices are regularly used in solving linear programming problems, so it is a precondition-prerequisite to know the four-arithmetic operation between matrices. When doing the addition between matrices, add the figures with corresponding positions.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} + \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} A+E & B+F \\ C+G & D+H \end{bmatrix} \tag{2}$$

When doing the subtraction between matrices, subtract the figures with corresponding positions.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} - \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} A-E & B-F \\ C-G & D-H \end{bmatrix} \tag{3}$$

The multiplication between matrices only exists when the number of columns of the first matrix is equal to the number of rows of the second matrix.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} AE+BG & AF+BH \\ CE+DG & CF+DH \end{bmatrix} \tag{4}$$

When dividing a matrix, there is a formula to calculate the inverse of it.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \frac{1}{|AD-BC|} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \tag{5}$$

2.4 Feasible solution

The problem is:

$$\text{Maximize the value of: } c^T x \tag{6}$$

$$\text{Among all vectors } x \in \mathbb{R}^n \text{ satisfying: } Ax \leqslant B \tag{7}$$

where $c \in \mathbb{R}^n$, $b \in \mathbb{R}^n$ are given vectors and A is a given $m \times n$ real matrix. The relation $\leqslant$ holds for the two vectors of equal length if and only if it holds componentwise. Any vector $x \in \mathbb{R}^n$ satisfying all constraints of a given linear program is a feasible solution [11].

2.5 Basic Feasible Solution

For the functions:

$$A\vec{x} = \vec{b}, \vec{x} \geqslant 0, m \leqslant n \tag{8}$$

$$\begin{bmatrix} a_{11}x_1 + & \cdots & +a_{1n}x_n = b_1 \\ \vdots & \ddots & \vdots \\ a_{mn}x_n + & \cdots & +a_{mn}x_n = b_m \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} x_1 \geqslant 0 \\ \vdots \\ x_n \geqslant 0 \end{bmatrix} \tag{10}$$

After combining all the equations, the eventually vertex is called basic feasible solution.

Example 1. Find all basic feasible sets of the following linear program (for example B = {1,4} is a basic feasible set):

$$\text{maximize } z = 6x_1 + 8x_2 + 5x_3 + 9x_4 \tag{11}$$

subject to,

$$x_1 + x_2 + x_3 + x_4 = 1 \tag{12}$$

$$x_1 - 3x_2 - 3x_3 + 2x_4 = 2 \tag{13}$$

with

$$x_1, x_2, x_3, x_4 \geqslant 0 \tag{14}$$

This is a problem in standard equational form. The feasible region can be described in matrix form as Ax=b, where:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -3 & -3 & 2 \end{bmatrix},\ b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{15}$$

Together with the non-negativity conditions x ≥0. This problem meets the standard conditions: the rows of A are independent (and thus rank A = 2), and there exists at least one solution x ∈ R4 (not necessarily feasible x ≥0) to Ax=b. This is a table with every possible solution (Table 1):

Table 1. Possible solutions of Example 1

Basic set	Nonbasic set	Solution	Feasible
x_1 , x_2	x_3 , x_4	($\frac{5}{4}$, - $\frac{1}{4}$, 0, 0)	Infeasible
x_1 , x_3	x_2 , x_4	($\frac{5}{4}$, 0, - $\frac{1}{4}$, 0)	Infeasible
x_1 , x_4	x_2 , x_3	(0, 0, 0, 1)	Feasible
x_2 , x_3	x_1 , x_4	-	-
x_2 , x_4	x_1 , x_3	(0, 0, 0, 1)	Feasible
x_3 , x_4	x_1 , x_2	(0, 0, 0, 1)	Feasible

Substitute the figures into the given formula, when basic set is B = {1,2}:

$$\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}^{-1} = \frac{1}{|1\times(-3)-1\times 1|}\times\begin{bmatrix} -3 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} \\ -\frac{1}{4} \end{bmatrix} \tag{19}$$

When basic set is $B = \{1,2\}, x_2 = -\frac{1}{4} < 0$, so this solution is infeasible.

When basic set is $B = \{1,3\}$:

$$\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}^{-1} = \frac{1}{|1\times(-3)-1\times 1|}\times\begin{bmatrix} -3 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} \\ -\frac{1}{4} \end{bmatrix} \tag{23}$$

When basic set is $B = \{1,3\}, x_3 = -\frac{1}{4} < 0$, so this solution is infeasible.

When basic set is $B = \{1,4\}$,

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{|1\times 2-1\times 1|}\times\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \tag{26}$$

$$\begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (27)$$

When basic set is $B = \{1,4\}, x_1 = 0, x_2 > 0$, so this solution is feasible.

When basic set is $B = \{2,3\}$:

$$\begin{bmatrix} 1 & 1 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (28)$$

$$\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} 1 & 1 \\ -3 & -3 \end{bmatrix}^{-1} = \frac{1}{|1\times(-3) - 1\times(-3)|} \times \begin{bmatrix} -3 & 3 \\ 1 & 1 \end{bmatrix} \quad (30)$$

Since $|1\times(-3) - 1\times(-3)| = 0$, this equation is meaningless.

When basic set is B = {2,4}:

$$\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (31)$$

$$\begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}^{-1} = \frac{1}{|1\times 2 - 1\times(-3)|} \times \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix} \quad (33)$$

$$\begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (34)$$

When basic set is $B = \{2,4\}, x_2 = 0, x_4 > 0$, so this solution is feasible.

When basic set is $B = \{3,4\}$:

$$\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (35)$$

$$\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (36)$$

$$\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}^{-1} = \frac{1}{|1\times 2 - 1\times(-3)|} \times \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix} \quad (37)$$

$$\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (38)$$

When basic set is $B = \{3,4\}, x_3 = 0, x_4 > 0$, so this solution is feasible.

To sum up, there are three basic feasible solutions in this problem.

3. Simplex Tableau Method

Example 2. Consider the following linear programming problem:

maximize $\quad z = 6x_1 + 8x_2 + 5x_3 + 9x_4 \quad (39)$

subject to:

$$2x_1 + x_2 + x_3 + 3x_4 \leqslant 5 \quad (40)$$

$$x_1 + 3x_2 + x_3 + 2x_4 \leqslant 3 \quad (41)$$

$$x_1, x_2, x_3, x_4 \geqslant 0 \quad (42)$$

Since the problem is to maximize z, it is easier to transfer it into a minimization problem. So:

minimize $\ z = -6x_1 - 8x_2 - 5x_3 - 9x_4 \quad (43)$

Let,

$$2x_1 + x_2 + x_3 + 3x_4 + x_5 = 5 \quad (44)$$

$$x_1 + 3x_2 + x_3 + 2x_4 + x_6 = 3 \quad (45)$$

Table 2. First step of Simplex Tableau Method

		x_1	x_2	x_3	x_4	x_5	x_6	
		-6	-8	-5	-9	0	0	
0	x_5	2	1	1	3	1	0	5
0	x_6	1	3	1	2	0	1	3
		6	8	5	9	0	0	0

The origin of Table 2 is filling-in the coefficient of each unknown number. When filling-in the last line, multiply each number in line 3 to the coefficient of x_5, add to the product of each number in line 4 to the coefficient of x_6, then minus each number in line 2, for example:

$$\begin{cases} 2\times0+1\times0-(-6)=6 \\ 1\times0+3\times0-(-8)=8 \\ 1\times0+1\times0-(-5)=5 \\ 3\times0+2\times0-(-9)=9 \\ 1\times0+0\times0-0=0 \\ 0\times0+1\times0-0=0 \end{cases} \quad (46)$$

Since $5\times0+3\times0=0$, the number in lower right corner is 0.

The next step is to find pivot. Pivot is the crossover point of the maximal positive number of line 5, and the minimal non-negative quotient of column 9 and the number with the maximal positive number of line 5. For example, the maximal positive number of line 5 is 9,

$$\begin{cases} 5¡Â3=\frac{5}{3} \\ 3¡Â2=\frac{3}{2} \\ \frac{5}{3}>\frac{3}{2} \end{cases} \quad (47)$$

Choose the smaller one, $\frac{3}{2}$. So the pivot is 2. So change x_6 into x_4.

Table 3. Second step of Simplex Tableau Method

		x_1	x_2	x_3	x_4	x_5	x_6	
	x_4	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{3}{2}$
	x_5	$\frac{1}{2}$	$-\frac{7}{2}$	$-\frac{1}{2}$	0	1	$-\frac{3}{2}$	$\frac{1}{2}$
		$\frac{3}{2}$	$-\frac{11}{2}$	$\frac{1}{2}$	0	0	$-\frac{9}{2}$	$-\frac{27}{2}$

The aim is to make the pivot be 1, so divide the whole row with the same number (Table 3). For example, the pivot $2¡Â2=1$, so:

$$\begin{cases} 1¡Â2=\frac{1}{2} \\ 3¡Â2=\frac{3}{2} \\ 1¡Â2=\frac{1}{2} \\ 0¡Â2=0 \\ 1¡Â2=\frac{1}{2} \end{cases} \quad (48)$$

Next, let the sum of the changed number and each number in the same column be 0. For example, the pivot $1\times(-3)+3=0$, so:

$$\begin{cases} \frac{1}{2}\times(-3)+2=\frac{1}{2} \\ \frac{3}{2}\times(-3)+1=-\frac{7}{2} \\ \frac{1}{2}\times(-3)+1=-\frac{1}{2} \\ 0\times(-3)+1=1 \\ \frac{1}{2}\times(-3)+0=-\frac{3}{2} \\ \frac{3}{2}\times(-3)+5=\frac{1}{2} \\ \frac{3}{2}\times(-9)+0=-\frac{27}{3} \end{cases} \quad (49)$$

The maximal positive number in line 4 is $\frac{3}{2}$, the minimal non-negative quotient of each line is $\frac{1}{2}¡Â\frac{1}{2}=1$.

Table 4. Third step of Simplex Tableau Method

		x_1	x_2	x_3	x_4	x_5	x_6	
	x_1	1	–7	–1	0	2	–3	1
	x_4	0	5	1	1	-1	2	1

		0	5	2	0	-3	0	–15

So, multiply each number in line 3 by 2 to make sure the pivot is equal to 1, then add the new number to let the number of the same column becomes 0 (table 4, 5 and 6).

Table 5. Fourth step of Simplex Tableau Method

		x_1	x_2	x_3	x_4	x_5	x_6	
	x_1	1	0	$\frac{2}{5}$	$\frac{7}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	$\frac{12}{5}$
	x_2	0	1	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$
		0	0	–1	-1	-2	–2	–16

Table 6. Fifth step of Simplex Tableau Method

		x_1	x_2	x_3	x_4	x_5	x_6	
-6	x_1	1	–2	0	1	1	–1	2
-5	x_4	0	5	1	1	-1	2	1
		0	–5	0	-2	-1	–4	–17

So $Z_{max}=17$, (2, 0, 1, 0, 0, 0). It is simpler to solve an optimizing problem by simplex tableau method, it only needs a few steps of calculation to help find the best feasible solution, which is an advantage of simple algorithm.

4. Conclusion

This research aims to help people making decision when facing an optimization problem. Simplex algorithm can also be used in interdisciplinary applications. For example, with the quick development of Artificial Intelligence, simplex algorithm can integrate with computer science for better efficiency in problem solving and fixing more complicated problems. To make simplex algorithm a stronger solver, it can be combined with other optimization algorithms like genetic algorithm and particle population algorithm. The future of simplex algorithm can be showed at efficiency improvement, degradation phenomenon treatment, and stability enhancement, scope of application expansion, algorithm fusion and deepening theoretical research. Being a significant method for linear programming, simplex algorithm will go on contributing to scientific and physical problems with its wide applicability. With the continuous progress of science and technology and the development of the application field, the simplex algorithm will continue to play an important role in optimization fields, and provide more effective solutions to solve physical problems.

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