ISSN 2959-6157

Application of Linear Programming in Automatic Pricing and Replenishment Strategies for Vegetable Commodities

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Abstract:

Vegetable commodities are indispensable parts of the current consumer market. Vegetables possess different characteristics such as seasonality, perishability and so on. Thus, it is critical to formulate reasonable automatic pricing and replenishment strategies for vegetable commodities. However, previous kinds of literature have either ignored some of the attributes of vegetables or restricted their application regions to some specific situations. This review compares the pros and cons of diverse linear programming methods and models in previous research, synthesizes the core method, and finally concludes with two relatively reasonable algorithms for pricing and replenishment strategies. It also gives an evaluation of these two algorithms and proposes suggestions for improvement. The purpose of this review is to provide guidelines for subsequent research to optimize existing methods and models to work out more objective and accurate optimal strategies. Ultimately, vegetable sales and supply can better meet the market demand, which can help rejuvenate economic vitality and bring in higher profits at the same time.

Keywords: Linear programming; pricing strategy; replenishment strategy; vegetable commodities.

1. Introduction

The booming social economy and improving living quality led to sustained and active growth in the consumer market in China. Fresh commodities are the rigid demand for consumer products that can safeguard people's livelihood and maintain social stability. What's more, vegetables are indispensable fresh commodities in people's daily lives, with nutrients beneficial to human health. Therefore, the market scale for fresh commodities is huge. However, vegetables possess natural attributes such as periodicity, seasonality, perishability, and vulnerability, coupled with different purchase costs caused by varieties and different origins of vegetables. Thus, formulating reasonable pricing and replenishment strategies is significant for greengrocers.

With better automatic pricing and replenishment strategies, vegetable sales can better meet the market demand and achieve good economic and social benefits, which is beneficial to rejuvenate business vitality and bring in higher profits at the same time. However, Previous research has simply focused on formulating pricing strategies or replenishment strategies in some specific situations or ignoring some attributes of vegetables [1-3]. For example, some research strategies do not take vegetables' attributes like seasonality and periodicity into account and can only be applied to a particular time context [1]. Other research strategies are limited to small and medium size shopping malls, ignoring supermarkets [2]. In conclusion, these strategies lack universality.

This review synthesizes methods and models of linear programming's application in automatic pricing and replenishment strategies for vegetable commodities from previous research. By comparing their strengths and weaknesses, it points out the research gap in the current area. The purpose of this review is to compare different influencing factors in vegetable commodities and to provide guidelines for future research to find out more objective and accurate optimal strategies.

2. Methodology

2.1 General Explanation

This review generally summarizes the method of linear programming for automatic pricing and replenishment strategies for vegetable products. The objective function (usually the profit function) and its constraints can be converted into a linear programming problem in the standard form. The standard form is as follows:

$$\begin{aligned} &min \quad z = c^T x, \\ &s.t. \quad Ax = b, \\ &\qquad\quad x \geqslant 0, \end{aligned} \tag{1}$$

where $A \in \mathbf{R}^{m \times n}, b \in \mathbf{R}^m, c \in \mathbf{R}^n$, $\mathrm{r}(A) = m$, $\mathrm{r}(A)$ is the rank of matrix A.

Usually, the final goal of solving this kind of problem is to work out an optimal feasible solution for the objective function. Before finding the optimal feasible solution, we need to calculate an initial feasible solution. The standard form needs to be transformed into the canonical form further.

Assume B as a sub-square matrix A of rank m, which is a basis. m linearly independent column vectors in matrix B are base vectors. Similarly, m components of variable x corresponding to the basis are basic variables, and the rest of the components are non-basic variables. Denote the sub-matrix of matrix A as N, removing the columns of the basis matrix B. N is called a non-basis matrix. Transform c and x into corresponding blocks, $c = \begin{pmatrix} c_B \\ c_N \end{pmatrix}, x = \begin{pmatrix} x_B \\ x_N \end{pmatrix}$.

Then, the standard form can be converted to the canonical form:

$$\begin{cases} min \quad z = c_B^T B^{-1} b - \left(c_B^T B^{-1} N - c_N^T \right) x_N, \\ s.t. \quad x_B + B^{-1} N x_N = B^{-1} b, \\ \qquad\quad x_B \geqslant 0, x_N \geqslant 0, \end{cases} \tag{2}$$

The canonical form is characterized by that the objective function simply contains non-basic variables and basic variables in the constraint matrix correspond to the unit matrix. If all the non-basic variables take the value of zero, which means $x_N = 0$, the solution obtained $x = \begin{pmatrix} x_B \\ x_N \end{pmatrix} = \begin{vmatrix} B^{-1}b \\ 0 \end{vmatrix}$ is called the basic solution.

When $B^{-1}b \geqslant 0$, the basic solution x becomes the basic feasible solution, and the corresponding base B becomes the feasible base.

2.2 Simplex Algorithm

Simplex algorithm is used as a general method to work out the optimal feasible solution in linear programming problems. For the convenience of calculation, a simplex table can be listed. Table 1 shows the initial simplex table.

Table 1. Initial simplex table

z	0	0	$\cdots$	0	σ_{m+1}	σ_{m+2}	$\cdots$	σ_n	$z^{(0)}$
x_1	1	0	$\cdots$	0	$a'_{1,m+1}$	$a'_{1,m+2}$	$\cdots$	a'_{1n}	b'_1
x_2	0	1	$\cdots$	0	$a'_{2,m+1}$	$a'_{2,m+2}$	$\cdots$	a'_{2n}	b'_2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_m	0	0	$\cdots$	1	$a'_{m,m+1}$	$a'_{m,m+2}$	$\cdots$	a'_{mn}	b'_m

In the first line, fill in the coefficients of the objective function in the canonical form (Let coefficients of all basic variables be 0 since the coefficients do not contain basic variables) $0, 0, \cdots, 0, \sigma_{m+1}, \sigma_{m+2}, \cdots, \sigma_n$; Then, list the constant term of the objective function $z^{(0)} = c_B^T B^{-1} b$ in the table. From Table 1, an initial basic feasible solution $X^{(0)} = (b_1', b_2', \cdots, b_m', 0, \cdots, 0)^T$ and the corresponding objective function value $z^{(0)}$ can be found clearly.

In Table 1, $\sigma_j (j = m+1, \cdots, n)$ are check numbers. After calculating check numbers, the optimality test can be performed to verify each solution's optimality. If the test fails to prove, then the basic feasible solution needs replacing, and repeated iterative operation will work out the final optimal feasible solution. In the process of concluding a feasible solution, this method utilizes the principle that the optimal solution of a linear programming problem corresponds to the vertices of the feasible domain.

For some special cases, if the original problem does not possess an obvious initial feasible basis, m slack variables need to be introduced as an auxiliary problem. Then, it can be transformed to a standard form again as follows:

$$\begin{aligned} &min \quad g = I x_a, \\ &s.t. \quad Ax + x_a = b, \\ &\qquad\quad x, x_a \geqslant 0, \end{aligned} \tag{3}$$

Denote the original problem's feasible region as D and the auxiliary problem's feasible region as D'. $x \in D$ and $\begin{pmatrix} x \\ 0 \end{pmatrix} \in D'$ are equivalent obviously. When $\begin{pmatrix} x \\ 0 \end{pmatrix} \in D'$ ∈ D' exists, if and only if min g = 0, it is allowed to obtain the initial solution by solving the auxiliary problem.

3. Method A

3.1 Method Introduction

Convert strategies into a linear programming problem by constructing an objective function with reasonable constraints to maximize profits. By calculating the predicted daily sales and unit cost of each category of vegetable, the optimal feasible solution can be worked out by the simplex algorithm.

3.2 Construct Proper Models to Predict Daily Sales

Firstly, Draw ACF (Auto-correlation Function) diagrams to analyze the correlation between time series data and different time lags for each vegetable category [4]. It means to calculate total sales auto-correlation coefficients (covariances) for each category of vegetables between January and other months in order. Below is the covariance formula to calculate total sales auto-correlation coefficients between January and other months:

$$\rho_{xy} = \frac{COV(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \quad (4)$$

Where x is one day's total sales in January and y is the corresponding one day's total sales in other months.

Then, models can be chosen to predict vegetables' daily sales in the coming week according to the trend of vegetables' ACF diagram in each category. Winters' multiplicative model and seasonal model can be built to predict the daily sales in the next week [5,6]. The equations in Winters' multiplicative model are the horizontal smoothing equation, trend smoothing equation, seasonal smoothing equation, and prediction equation respectively:

$$\begin{aligned} l_t &= \alpha \frac{x_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1}), \\ b_t &= \beta(l_t - l_{t-1}) + (1-\beta) b_{t-1}, \\ s_t &= \gamma \frac{x_t}{l_{t-1} - b_{t-1}} + (1-\gamma) s_{t-m}, \\ \hat{x}_{t+h} &= (l_t + h b_t) s_{t+h-m(k+1)}, k = \left[\frac{h-1}{m}\right], \end{aligned} \quad (5)$$

The equations in the seasonal model are the horizontal smoothing equation, seasonal smoothing equation, and prediction equation respectively:

$$\begin{aligned} l_t &= \alpha(x_t - s_{t-m}) + (1-\alpha) l_{t-1}, \\ s_t &= \gamma(x_t - l_{t-1}) + (1-\gamma) s_{t-m}, \\ \hat{x}_{t+h} &= l_t + s_{t+h-m(k+1)}, k = \left[\frac{h-1}{m}\right], \end{aligned} \quad (6)$$

where m is cycle length, α is horizontal smoothing parameters, β is trend smoothing parameter, γ is seasonal smoothing parameters, is time raw value, and is prediction in period h.

3.3 Calculation of Unit Cost

Calculate the unit cost of each category of vegetable.

$$B = C \times (1 + L) \quad (7)$$

where B is unit cost, C is wholesale price, and L is depletion rate.

3.4 Linear Programming for Working Out Strategies

The most significant method is to establish an objective function that maximizes profits. The objective function and its constraints are as follows:

$$\max \sum_{i=1}^{6} \sum_{j=1}^{7} S_{ij} \times (P_{ij} - B_i) - R_{ij} \times B_i \times L_i$$

$$st. \begin{cases} S_{ij} \leqslant R_{ij} \leqslant S_{imax_i}, i = 1,2,\cdots 6; j \leqslant 1,2,\cdots,7 \\ 110\% \times B_i \leqslant P_{ij} \leqslant 150\% \times B_i, i = 1,2,\cdots 6; j \leqslant 1,2,\cdots,7 \\ B_i \times R_{ij} \times L_i + (R_{ij} - S_{ij}) \leqslant L_i \times S_{imax}, i = 1,2,\cdots 6; j \leqslant 1, \end{cases} \quad (8)$$

Constraints: the daily supply for each category of vegetables should not be less than the daily sales volume; each type of vegetable's future daily replenishment should not be more than the maximum daily sales volume of each category in the previous three years; pricing for each category of vegetables should not be less than 110% or more than 150% of unit cost; depletion on daily replenishment should not be more than the maximum daily average depletion of each category of vegetables in the previous three years.

where S is daily sales volume, R is daily replenishment volume, P is pricing, B is unit cost, and L is average depletion rate. The final optimal solution for pricing and replenishment can be worked out by the simplex algorithm.

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4. Method B

4.1 Method Introduction

Fill the vacant data and do data analysis. Build a LightGBM Sales Forecasting Model to predict vegetable sales [7,8]. Then, take advantage of the least squares method to find out the functional relation between total daily sales and replenishment for fresh products. Finally, the optimal pricing and replenishment strategies can be worked out.

4.2 Analyses Before Building the Model

4.2.1 Sliding time window analysis

Sliding time window analysis can be used to make up for the vacant data on total daily sales for each category of vegetable. Denote the data time intervals as $[T_1, T_2, \cdots, T_n]$ and assume the time intervals are the same length. Suppose $[T_i, T_{i+1}, T_{i+2}, \cdots, T_{i+n}]$ to be a sliding window. Figure 1 shows that the data will be updated to fill in the missing data as the window slides to the subsequent time interval.

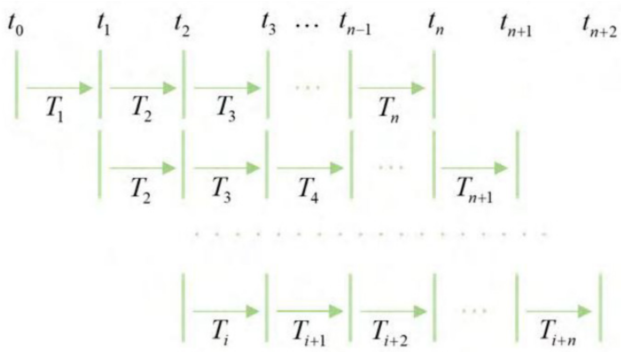

Fig. 1 Explanation diagram of Sliding time window [7]

where i is a natural number, t_i represents a time point, T_i stands for the time interval between t_{i-1} and t_i.

4.2.2 FCM algorithm analysis

By using the FCM (fuzzy c-means) algorithm, the core method is to separate n data points into k fuzzy clusters and discover each cluster's center, in order to minimize the value of the objective function [9, 10]. Suppose $X = \{x_1, x_2, \cdots, x_n\}$ to be n data samples; suppose c to data sample classifications' number in the constraint $2 \leqslant c \leqslant n$; assume $\{A_1, A_2, \cdots, A_C\}$ as the corresponding c classifications; assume U as the similarity classification matrix; let $\{v_1, v_2, \cdots, v_c\}$ be the clustering center of each category; let $\mu_k\ (x_i)$ be sample x_i ' s degree of affiliation to class A_k (abbreviated as μ_{ik}). Finally, J_b can be obtained as the objective function in the below. The goal of the FCM algorithm is to discover an optimal classification that outputs the minimum value j_b. When the minimum loss is reached, each class's clustering centers and each sample's degree of affiliation will be obtained.

$$J_b(\mathbf{U}, v) = \sum_{i=1}^{n} \sum_{k=1}^{c} (\mu_{ik})^b (d_{ik})^2, \quad (9)$$

$$d_{ik} = d(x_i - v_k) = \sqrt{\sum_{j=1}^{m} (x_{ij} - v_{kj})^2}, \quad (10)$$

where b is a weighting number with constraint $1 \leqslant b < \infty$, d_{ik} is the distance between x_i and the center of class k in sample i , m is the sample features' quantity,

$$\sum_{j=1}^{c} \mu_j(x_i) = 1, i = 1, 2, \cdots, n.$$

4.2.3 SWOT analysis

Based on SWOT analysis [11], it is possible to simulate the competitive strengths and weaknesses, opportunities, and threats in the market, which can help explore the factors affecting the pricing of vegetables in the market. According to temporal features that have been obtained, it can be analyzed that there is a relationship between price and supply. It means that the relationship function can be established by using the least square method.

4.3 Use LightGBM Sales Forecasting Model and Work out the Strategies

Establish the function to predict daily sales by using the LightGBM Sales Forecasting Model:

$$y = b + w_1 * x + w_2 * x^2 + \ldots + w_n * x^n, \quad (11)$$

where x is the volume of daily sales for each type of vegetable, y is the daily average price for each type, w_i is the coefficient of the polynomial, and b is the intercept.

The coefficients of the polynomial can be adjusted by using the least squares method to minimize the sum of the squares of residuals between data points and the fitting curve. Then, the problem can be converted to a function similar to a linear programming problem:

$$(w^*, b^*) = argmin \sum_{i=1}^{m} (f(x_i) - y_i)^2. \quad (12)$$

Calculate two parameters: w^* and b^* . The function can be solved by finding the polynomial's partial derivatives:

$$\frac{\partial E(w,b)}{\partial w} = 2\left(w \sum_{i=1}^{m} x_i^2 - \sum_{i=1}^{m} (y_i - b) x_i\right)$$
$$\frac{\partial E(w,b)}{\partial b} = 2\left(mb - \sum_{i=1}^{m} (y_i - wx_i)\right). \quad (13)$$

The optimal solution of w and b can be obtained when the equations above equal to 0. The pricing strategies can be worked out in the equations below:

$$w = \frac{\sum_{i=1}^{m} y_i (x_i - \bar{x})}{\sum_{i=1}^{m} x_i^2 - \frac{1}{m}\left(\sum_{i=1}^{m} x_i\right)^2}$$
$$b = \frac{1}{m} \sum_{i=1}^{m} (y_i - wx_i) \quad (14)$$

In conclusion, the optimal strategy for pricing can be obtained by substituting the predicted sales from the LightGBM Sales Forecasting Model into the polynomial equation and solving a linear programming problem under the least squares method. The replenishment strategy can be worked out by inputting price values into the relationship function between price and supply.

5. Conclusion

This review synthesizes methods and models in some previous research, by comparing their advantages and disadvantages, to find an optimal strategy for automatic pricing and replenishment of vegetable commodities. Previous research pays less attention to the demand and supply of vegetables at different periods from an overall perspective or ignores some of the attributes of vegetables. Thus, this review summarizes two methods, which use more scientific and systematic mathematical approaches to study and analyze the vegetable market by considering production and marketing activities based on the actual situation. In method A, the algorithm process is simpler. For example, the ACF diagram is easy to draw; thus, the prediction of total sales for each category of vegetables is simpler. In method B, the solution is more accurate. For example, models used in this method are trained with big data, which makes the results more scientific.

However, there are still some limitations to the two methods mentioned in this review. In method A, using traditional vegetable categorization may produce some errors in the sales forecasting process. It means that it cannot ensure the sales of different types of vegetables under the same category are as similar as possible. In method B, both the Polynomial Fitting Model and LightGBM Sales Forecasting Model need testing, which makes the process longer. What's more, model training requires more steps and costs a lot of time.

This review hopes to provide future research with guidelines to conclude optimal strategies more objectively and accurately. Finally, the consumer market in vegetable commodities can achieve a balance between sales and supply, rejuvenate economic vitality, and bring in higher profits at the same time with improved automatic pricing and replenishment strategies.

Authors

Zhengyi Bai and Miao Chen are co-first authors of this article.

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