ISSN 2959-6157

The Application of Calculus in Physics

Zihao Ge

Washington Academy, East Machias, United States
*Corresponding author: gez2025@gmail.com

Abstract:

One of the foundational areas of mathematics is calculus. Functions, limits, differential and integral calculus, and their applications make up most of the subject. Differentiation and integration are the limits of certain processes and forms, functions are the essential subjects of calculus investigation, and limits are the fundamental notions of calculus. This article explores the applications of calculus to gravity, rotational inertia, and force in physics. This exploration of calculus highlights the enduring importance of this mathematical tool in theoretical and applied physics, and its key role in driving future scientific breakthroughs. Calculus is a mathematical tool for studying the rate of change, area under a curve, and limits. By precisely expressing the rate of change information of a function at a given location, such as slope, area, and curvature, it can aid in the understanding and solution of problems pertaining to changes in physics, engineering, biology, and other subjects. The growth of mathematics is substantially aided by the introduction of calculus. employing calculus to find solutions to a wide range of issues that usually required only elementary mathematics. It makes it possible to describe functions, velocities, accelerations, and curve slopes using a common set of symbols.

Keywords: Calculus; physics; application.

1. Introduction

Calculus, the mathematical study of continuous change, plays a pivotal role in modern physics. As a fundamental tool, it provides the framework for describing and analyzing physical phenomena that involve dynamic systems and variable quantities. Few simple instances of the application of calculus in physics could be calculating the gravitational force, inertia, energy, and etcetera.

Calculus is crucial for examining the connection between forces and motion in the study of gravitational force. The ancients realized that the planets and stars were all orbiting the sun when they studied the motions of the planets among them before the invention of the calculus. The principle of inertia, later developed by Galileo, asserts that an undisturbed item always moves in a straight line unless there is an external force acting on the system. Newton extended this principle by introducing the concept of force, which needs to be applied to the change the velocity or the motion path of an object. In 2008, Xin studied the applications of definite integrals in geometry and economics [1]. In 2010, Ren mainly used the idea of infinitesimal method to solve physical problems such as variable force work, water pressure, gravity, and moment of inertia [2]. In 2010, Liang analyzed the application of calculus in general physics, enabling students to quickly understand the ideas of calculus and proficiently use calculus methods to analyze physics problems [3]. In 2012, Zhang studied the applications of calculus in thermal power unit of power plant [4]. In 2013, Wang introduced the application of definite integrals in high school physics, using problems such as variable force work, two average values of force, and effective values of alternating current as examples [5]. In 2013, Wang infused the ideas and methods of calculus into high school physics teaching, Integrating the content of the college entrance examination and the independent enrollment examination, considering both aspects, to achieve the connection between high school physics and university general physics teaching [6]. In 2016, Ma conducted research on a class of kinematic problems related to acceleration and velocity [7]. In 2016, Tan elaborated on the application of calculus ideas and methods in physics concepts and theories [8]. In 2019, Chen studied the specific application of calculus ideas in middle school physics teaching [9]. In 2020, Wei conducted research on how to use calculus thinking to analyze high school physics problems and simplify complex physics problems [10]. This paper is organized as follows: the application of Calculus in gravitational force will be given in Section 2.1. Section 2.2 and 2.3 will be devoted to discussing the application of Calculus in inertia and force respectively.

2. Application of Calculus

2.1 Gravitational Force

Calculus was created to show how, in Newton's research of planet motion, the assumption that the forces acting on planets were precisely directed toward the sun led to the development of Kepler's second law, which asserts that equal regions should be swept out in equal timeframes. Newton ultimately asserted that the phenomena of gravitation was universal by using his inverse square law to demonstrate that the planet must move in accordance with Kepler's first law. Newton's law of universal gravitation became the name for it:

$$F = G\frac{m_1 m_2}{r^2}, G = 6.67430*10^{-11}\frac{N*m^2}{kg^2} \quad (1)$$

Example 1. Consider two identical spheres with mass of 15g. The distance between the two particle is 5cm. Calculate the gravitational force between the spheres.

Proof:

$$F = G*\frac{15*15}{5^2} \quad (2)$$

$$F = 9*G\frac{N*m^2}{kg^2} \quad (3)$$

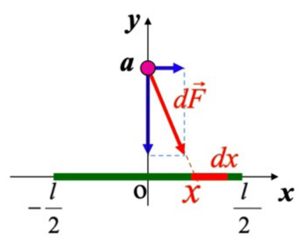

Fig 1: The graph of Example 2

Example 2. Consider a small mass element of length dx at a distance x from the centre of the rod as showed in Fig 1. Mass of the mass element, $dm = \left(\frac{M}{L}\right)*dx$. Derive an expression for the gravitational field.

Proof: From the symmetry, we know that the horizontal force Fx = 0, and the vertical component are:

$$dF_y = -G\frac{m*\frac{M}{l}dx}{\left(\sqrt{a^2+x^2}\right)}*cos\theta = -\frac{GaMm}{l\left(a^2+x^2\right)^{\frac{3}{2}}}dx \quad (4)$$

Therefore, $F_y = \int_{-\frac{1}{2}}^{\frac{1}{2}} -\frac{GaMm}{l\left(a^2+x^2\right)^{\frac{3}{2}}}dx = -\frac{2GMm}{a\sqrt{l^2+4a^2}}$.

The magnitude of the gravitational force for the thin straight rod on the particle is,

$$F = \sqrt{F_x^2+F_y^2} = \left|F_y\right| = \frac{2GMm}{a\sqrt{l^2+4a^2}} \text{ units of force.}$$

The gravitational force in terms of vector:

$$\vec{F} = \left\{0, -\frac{2GMm}{a\sqrt{l^2+4a^2}}\right\} \quad (5)$$

2.2 Inertia

As with translational motion, moment of inertia is a quantitative measure of rotating inertia if mass is a quantitative measure of linear inertia. Newton's First Law states that an object's mass increases with its inertia and resistance to changes in linear velocity. Greater moments of inertia are produced by more concentrated mass in rigid bodies and particle systems compared to objects and systems that have the same mass concentrated closer to the rotating axis. This is how the mass moment of inertia is written:

$$I = \int_m r^2 dm \quad (6)$$

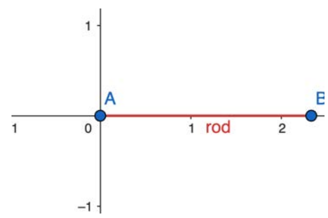

Fig 2: The graph of Example 3

Example 3. Calculate the moment of inertia of a thin rod with length l rotating around the y-axis in Fig 2.

Proof:

$$I = \int_0^l x^2 dx = \frac{l^3}{3} \text{ units of inertia}$$

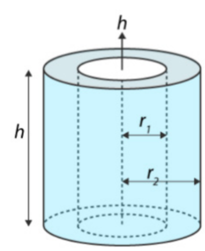

Fig 3: The graph of Example 4

Example 4. Derive the inertia formula of a hollow cylinder in Fig 3, where $m = pV; V = \pi hr^2$.

Proof:

$$dm = p*dV; dV = \pi h\left(2rdr\right)?dm = p\pi h\left(2rdr\right) \quad (7)$$

$$I = \int_{r_1}^{r_2} r^2 p\pi h\left(2rdr\right) = 2\pi ph\int_{r_1}^{r_2} r^3 dr = \frac{\pi ph}{2}\left[r_2^4 - r_1^4\right] \quad (8)$$

Derive the volume and mass formula of hollow cylinder to simplify the inertia formula

$$V = \pi hr_2^2 - \pi hr_1^2 = \pi h\left(r_2^2 - r_1^2\right) \quad (9)$$

$$m = pV = p\pi h\left(r_2^2 - r_1^2\right) \quad (10)$$

Simplify the inertia formula and substitute m from above into the formula

$$I = \frac{\pi ph}{2}\left[r_2^4 - r_1^4\right] = \frac{\pi ph}{2}\left[r_2^2 - r_1^2\right]\left[r_2^2 + r_1^2\right] \quad (11)$$

$$I = \frac{1}{2}m\left(r_2^2 + r_1^2\right) \quad (12)$$

2.3 Work and energy

In physics, work is the result of displacement multiplied by force. Both displacement and force are vectors with magnitude and direction. The components of force and displacement that move in the same direction are used to calculate work. The work performed by the force in the direction of displacement is given by the product of these two component vectors. In mathematics, the definition of work is

$$W = Fd \quad (13)$$

Since the force is practically constant over all lengths, if the force is applied vary, the mathematical computation is simply adding up the minuscule work that is done by pushing an object over a sequence of infinitesimal distances. It is therefore equal to determining the integral:

$$W = \int_{x_i}^{x_f} F_x dx \quad (14)$$

The force perpendicular to the motion does not act when the starting and ending points are the same, as long as x_i = x_f. The integral would be in the form of a line integral if the route of motion is not straight line:

$$W_{AB} = \int_A^B F cos\theta ds \quad (15)$$

Example 5. When a force of 20N pushes a 5.0 kg box across the floor at a steady speed, how much labor is required to move the box 7.5 meters?

Proof: $W = 20*7.5 = 150Nm$

Example 6. The function can be used to represent a variable force operating on an object, expressed in Newtons: $F(x) = x^2 - 4x + 7$, where x represents the separation from the origin in meters. In order to move the object from x = 1 to x = 4, how much work will it take?

Proof:

$$W = \int_A^B F(x)dx = \int_1^4 (x^2 - 4x + 7)dx \quad (16)$$

$$W = \left[\frac{x^3}{3} - \frac{4x^2}{2} + 7x\right]_1^4 = 12J \quad (17)$$

3 Conclusion

The basic idea of the definite integral infinitesimal method mainly includes replacing curves with straight ones and discarding high-order infinitesimal quantities. The specific implementation is divided into four steps: subdivision, approximate substitution, summation, and finding limits. With calculus, physicists can model dynamic systems, calculate instantaneous rates of change, and gain a more precise understanding of physical phenomena such as gravity, inertia, and energy. On the other hand, integral calculus allows the calculation of areas under curves, such as the work done by a force and the accumulation of physical quantities over time and space. The application of calculus to physics is more than just a theoretical exercise. It is a cornerstone of modern scientific research and has driven significant advances in our understanding of the natural world. Furthermore, continued research into the application of calculus to physics has the potential to unlock new discoveries, refine existing theories, and solve unsolved problems in the field. It not only advances our understanding of the physical universe, but also drives scientific progress and shapes the future of physics and its related fields.

References

[1] Xin C. Research on the application of definite integrals. Modern Commerce and Industry, 2008, 20 (11): 2.

[2] Ren J. A brief discussion on the Physical applications of definite integral. Forest Teaching, 2010, 10: 2.

[3] Liang X. Exploration of the application of Calculus in college Physics. Journal of Gansu Normal University, 2010,15(2): 3.

[4] Zhang X. The application of definite integral. Shang, 2012, 19: 2.

[5] Wang C. The application of definite integral in high school physics. Physics Bulletin, 2013, 3: 3.

[6] Wang Y. Research on the application of Calculus in high school Physics teaching. Journal of Middle School Physics Teaching, 2013, 10: 3.

[7] Ma Y. A simple application of calculus in high school physics. Mathematics, Physics, and Chemistry Learning: Grade 1 and 2, 2016.

[8] Tan M. The application of calculus ideas and methods in college physics teaching. Continental Bridge View, 2016, (16):2.

[9] Chen L. The application of micro element method in high school Physics learning. Teacher Education Forum, 2019, 4.

[10] Wei R. A brief discussion on the application of Calculus in middle school Physics. Guangxi Physics, 2020, 041(004):61-63.