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Comparative Study on the Application Efficacy of Small Disturbance Equation versus One-Dimensional Euler Equation in Nozzle Flow Analysis

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Abstract:

This paper presents a numerical simulation study of subsonic and supersonic nozzle flow regimes utilizing the Small Disturbance Equation (SDE), implemented through Python to analyze flow stability under various boundary conditions. The SDE, extensively applied in aerospace, meteorology, and fluid mechanics, offers a critical framework for examining aircraft stability and maneuverability, essential for ensuring flight safety. Additionally, its application extends to spacecraft stability and control in aerospace science. The findings from this study reveal that subsonic flows, characterized by their stability and smoothness, respond more predictably under varying boundary conditions compared to their supersonic counterparts. Conversely, supersonic flows demonstrate increased sensitivity to changes in boundary conditions, resulting in more complex flow patterns. This sensitivity underscores the need for precise control mechanisms in supersonic applications to maintain flow stability and ensure the safety and efficiency of aerospace operations. The simulations underscore the practical importance of the SDE in advancing the understanding of dynamic flow problems across different scientific and engineering disciplines.

Keywords: Nozzle flow, Small Disturbance Equation, Subsonic Flow, Supersonic Flow.

1. Introduction

Subsonic and supersonic nozzles are integral to advanced applications in aerospace engineering and aerodynamics, controlling fluid speed and pressure to achieve precise functional outcomes [1]. The kinetic energy transferred through the nozzle's internal design accelerates fluids, increasing their velocity and kinetic energy. This accelerated fluid experiences a decrease in velocity upon exiting the nozzle due to the conservation of kinetic energy, transforming kinetic energy back into pressure energy or other energy forms [2]. In aerospace, nozzles are crucial for jet and rocket engines, facilitating the acceleration of gases from combustion chambers to generate thrust [3][4].

Current Research Status: Nozzles also find critical applications in the energy sector, including in gas and steam turbines in the power generation industry, where they manage fuel and steam flow to enhance energy conversion efficiency [5]. Similarly, in nuclear reactors, nozzles control coolant flow, ensuring reactor operational safety and efficiency. The Small Disturbance Equation (SDE) plays a pivotal role in flight science, particularly in assessing the stability and maneuverability of aircraft. This equation, which presupposes minor perturbations in velocity fields, offers a simplified yet effective tool for analyzing flow disturbances [6][7]. The SDE's utility extends beyond complex equations like Navier-Stokes, providing reliable flow behavior predictions without extensive computational demands [8].

The focus of this study involves applying the Small Disturbance Equation to explore nozzle flows under various flow regimes, specifically subsonic and supersonic conditions. Python is utilized to solve the SDE, allowing for precise calculations of fluid streamlines under different boundary conditions. This approach facilitates a deeper understanding of the dynamics at play within different nozzle applications, emphasizing the behavior of fluid flows in both subsonic and supersonic scenarios. The exploration of these dynamics provides critical insights into the operational efficiencies and potential optimizations of nozzle designs used in high-demand aerospace and energy applications.

2. Research Methods and Processes

2.1 Theoretical Framework

In this study, a mathematical model was first constructed based on the Small Disturbance Equation, tailored to the specific geometry of the nozzle. Under the assumption of small flow disturbances, this equation simplifies the analysis of flow characteristics within the nozzle. Different flow velocities, including supersonic and subsonic, along with varying boundary conditions, were considered to ensure the comprehensiveness and accuracy of the analysis.

The Navier-Stokes Equations:

The Energy Equation.

$$\frac{\partial \rho}{\partial t}+\mathrm{div}\left(\rho U\right)=0 \qquad (1)$$

The Momentum Equation.

$$\frac{\partial\left(\rho U\right)}{\partial t}+\mathrm{div}\left(\rho U\otimes U+pI\right)=\mathrm{div}\tau \qquad (2)$$

The Continuity Equation.

$$\frac{\partial\left(\rho E\right)}{\partial t}+\mathrm{div}\left(\rho EU\right)=\mathrm{div}\left(k\nabla T\right)+\mathrm{div}\left(\sigma U\right) \qquad (3)$$

The three fundamental formulas in fluid mechanics, often known as the continuity equation, represent core principles:

Conservation of Mass: This principle asserts that the mass within a system moving with the fluid remains constant. The continuity equation for mass conservation can be derived using the divergence theorem, which plays a crucial role in fluid mechanics by converting volumetric integrals into surface integrals [9]. This conversion significantly reduces the computational burden, facilitating more efficient calculations in practical applications.

Application of the Divergence Theorem in One Dimension: In one-dimensional systems, the divergence theorem takes a discrete form. This is represented in a staggered grid where U is positioned at one edge of a vertical unit and V at a horizontal edge. Through this discrete divergence operator, internal contributions negate each other, isolating only the boundary terms. This simplification underscores the theorem's utility in reducing complex calculations to more manageable boundary-focused evaluations.

Conservation of Energy: Defined as the total of internal and kinetic energies, energy conservation is influenced by the work done by surface forces and heat flux. The energy equation, derived via the divergence theorem, embodies this principle. It illustrates how energy changes within a system are influenced by the interaction of surface forces and heat flow.

The Euler Equations.

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho u\\ \rho E\end{pmatrix}+\frac{\partial}{\partial x}\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho u\left(E+p/\rho\right)\end{pmatrix}=0 \quad p=\left(\gamma-1\right)\rho e \quad c^2=\gamma p/\rho \qquad (4)$$

A simplification of the full potential equations is done in the case of thin obstacles, such as thin airfoils. The discussion to 2D. Since the obstacle is "small" it's effect on the flow is small and consider perturbation to uniform flow with velocity of magnitude U_∞ in the x-direction. The potential has a representation $\varphi=U_\infty\left(x+\Phi\right)$.

Where velocities are recovered from the potential using the formulas.

$$u=U_\infty\left(1+\Phi_x\right) \quad v=U_\infty\Phi_y \qquad (5)$$

The equation becomes $\left(1-M_x^2\right)\Phi_{xx}+\Phi_{yy}=0$ which can be simplified further to:

$$\left(1-M_\infty^2\right)\Phi_{xx}+\Phi_{yy}=0 \qquad (6)$$

The wall BC becomes $v=\left(U_\infty+u\right)\dot{f}\left(x\right)\approx U_\infty\dot{f}\left(x\right)$ where f(x) is the shape of the airfoil.

These equations need to be supplied with a domain, boundary conditions, initial conditions. For stationary problems set time derivative to 0.

2.2 Boundary Condition Settings

To explore the impact of various boundary conditions on flow dynamics, different scenarios were established, featuring varied inlet velocities and outlet pressures in a nozzle setup. Simulations were conducted across both supersonic and subsonic flow regimes, with adjustments to boundary conditions tailored to assess their effects on the flow field. The boundary conditions were also modified to correspond with the shape of the obstacle to better suit the flow characteristics. The Successive Over-Relaxation (SOR) method was employed, effectively reducing the error incrementally through each iteration of the equation. This approach helps in achieving a more accurate representation of flow behavior under diverse conditions.

2.3 Numerical Simulation

For the numerical solution, Python was utilized to solve the Small Disturbance Equation (SDE). Originally developed for turbomachinery applications, the SDE Computational Fluid Dynamics (CFD) has been extended to external aerodynamics. The Reynolds averaged Navier Stokes (RANS) equation was also addressed using SDE CFD to tackle related issues, enhancing the precision and speed of the Doublet-Lattice Method, especially for complex devices. This is achieved through the use of linearization techniques to reduce simulation costs. For instance, when compared to the Navier-Stokes (NS) method alone, the computation time for the small disturbance NS method is significantly reduced, by up to half of an order of magnitude [10].

The SDE method's growing refinement facilitates its application to numerous practical engineering problems, including the calculation of dynamic stability derivatives and the research into aeroelastic and aeroservoelastic behaviors, as well as the creation of reduced-order models.

In coding, a five-point difference scheme was employed to discretize the equation. By calculating the error and optimizing iteratively, the error was reduced to $\left(10^{-6}\right)$, achieving higher accuracy. To begin solving the small disturbance equation, it is first necessary to discretize it due to the complex nature of the direct solution process. The divergence theorem is crucial in this step as it transforms a body integral into a surface integral, simplifying the computational effort significantly. In one-dimensional applications, the theorem adopts a discrete form, represented by a staggered grid where U and V are positioned at opposite edges of a unit cell. This discrete divergence operator reveals that all internal contributions negate each other, leaving only the boundary terms to be considered.

By employing the finite difference method, the equation was discretized and then solved iteratively. Special attention was given to the influence of boundary condition changes on the results, with multiple simulations conducted under different initial and boundary conditions to generate corresponding streamline diagrams.

3. Results and Analysis

3.1 Subsonic Flow Results

This image represents the streamlines of subsonic flow within the nozzle. The streamlines are relatively uniform and parallel, indicating stable and smooth flow throughout the region. This suggests that the subsonic flow is stable and well-behaved, with no significant shocks or discontinuities. This is a typical characteristic of subsonic flow, where the Mach number is less than 1, and the flow is primarily incompressible. As show in the fig.1.

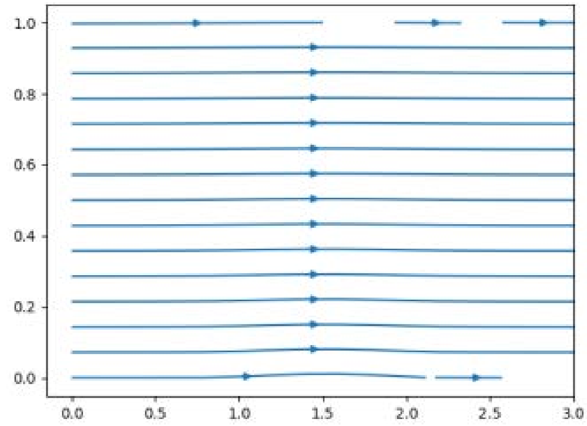

Fig. 1 Subsonic Boundary 1 (Photo credit: Original).

This image displays the streamlines of subsonic flow under a different boundary condition. It is evident that the streamline pattern shows significant changes near the inlet and outlet, which may indicate higher velocity gradients or pressure changes in these regions. As show in the fig.2.

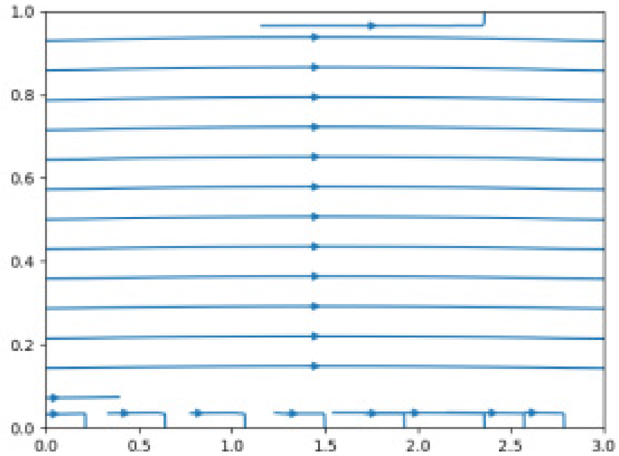

Fig. 2 Subsonic Boundary 2 (Photo credit: Original).

3.2 Supersonic Flow Results

This image shows the streamlines of supersonic flow. The streamlines exhibit more curvature and compression, particularly near the centerline, indicating the presence of shock waves or compression regions. In supersonic flow, the Mach number is greater than 1, leading to compressibility effects and shock formation. As show in the fig.3.

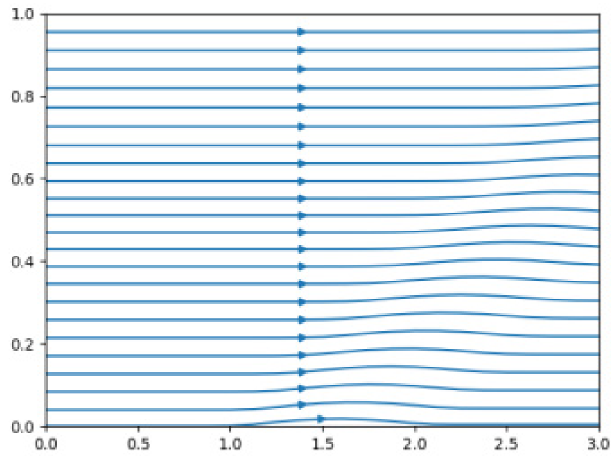

Fig. 3 Supersonic Boundary 1 (Photo credit: Original).

The vector plot indicates that the flow is experiencing significant disturbances, possibly due to the interaction of shock waves with boundaries or other flow features. In such cases, even small changes in conditions can lead to large variations in the flow field. As show in the fig.4.

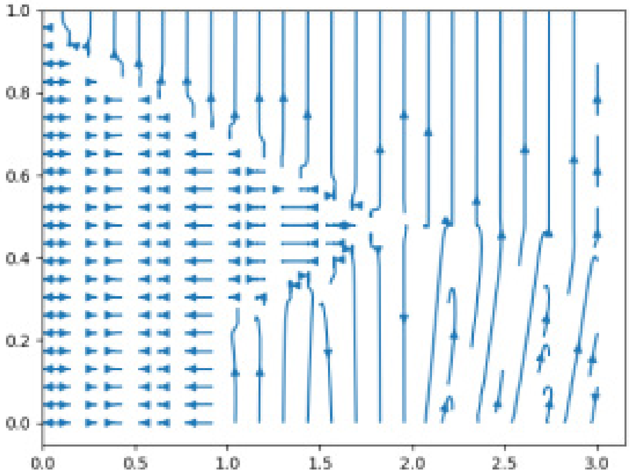

Fig. 4 Supersonic Boundary 2 (Photo credit: Original).

3.3 Comparative Results and Analysis

Subsonic Flow: The results for subsonic flow indicate stable, smooth flow with slight variations depending on boundary conditions. This aligns with the expected behavior for flows with a Mach number less than 1.

Supersonic Flow: The supersonic flow results highlight the complexity and sensitivity of such flows to boundary conditions, with evident shock formations and significant variations in the flow field.

4. Conclusion

This study has successfully employed the Small Disturbance Equation to analyze and simulate the dynamics of subsonic and supersonic flows using Python, demonstrating the equation's effectiveness in delineating the distinct behaviors of fluids under varying boundary conditions. The analysis reveals that subsonic flows exhibit greater stability and smoothness, whereas supersonic flows respond with increased sensitivity and complexity to changes in boundary conditions. These results underscore the utility of the Small Disturbance Equation in capturing essential flow characteristics across different speed regimes, facilitating a deeper understanding of fluid dynamics in practical applications.

Limitations of the Study: Despite the successes highlighted, the study acknowledges several areas requiring further improvement. The simulation results indicate the presence of complex vortical structures and significant non-uniformities in the flow field, particularly in supersonic conditions. These anomalies may suggest potential numerical instabilities or challenges in physical modeling that need addressing. To enhance the model's accuracy and physical realism, future efforts could focus on optimizing boundary conditions, increasing grid resolution, and refining numerical methods. Additionally, it is crucial to monitor and minimize error metrics consistently throughout simulations; unresolved errors can compromise the reliability of the flow visualizations, making the streamlines potentially misleading. Optimizing these elements will strengthen the model's predictive capabilities and improve its applicability in engineering and aerodynamics.

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