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The Application of Calculus in Credit Default Swaps: Pricing, Risk Assessment, and Market Implications

Jiatai Lin

Creanlutheran High School, Irvine, CA, 92618, United States
Corresponding author: jiatai.lin.25@stu.creanlutheran.org

Abstract:

This paper aims at the investigation of one of the derivatives, namely from the group of Credit Default Swaps, with the help of calculus and, more particularly, differential equations and integrals. The plan is to show how these calculative instruments may be utilized to price and risk measure CDS and to thereby illustrate the utility of such models in mitigating financial risks. The paper is structured as follows: In Section 2, 'The mechanics and functionality of CDS', the focus is brought on the specifics of construction of CDS contracts, and the matter of their pricing. Valuation of CDS is described in section three where differential equations and integrals are used in the mathematical modeling of these instruments. Section 4 also focuses on the connection between CDS and market stability; this section also looks at the positive and negative effects of CDS on the financial markets. Finally, Section 5 provides conclusion and recommendation for the subsequent research.

Keywords: Credit Default Swaps (CDS), Financial Derivatives, Management Credit Risk

1. Introduction

Credit Default Swaps (CDS) appeared in the early nineties as the new financial tool which enabled financial parties to insure against losses owing to a default on borrowing. A CDS is essentially a contract between two parties: The protection buyer – a party that buys protection at a premium; and the protection seller – a party that agrees to pay back the protection buyer in case of an occurrence of default. The contract usually refers to a certain debt instrument, such as a corporate bond or sovereign bond, and credit events are such events as bankruptcy, non-payment or restructuring. First, developed as a hedging instrument for banks' credit risk, the CDS gained importance in the financial industry very soon. The market grew very quickly as CDS had applications not only in hedging but also in speculation – investors could take positions on the credit risk of various entities without being exposed to the instruments themselves. By the mid-2000s the net notional amount of outstanding CDS contracts stood at a number in the trillions of dollars and has become one of the largest markets in the world today. The importance of CDS in financial markets cannot be gained. They support the credit markets, facilitate better risk diversification and help institutions optimize their capital. But not less importantly, the usage of CDS has also brought new opportunities which contributed to the crisis in 2008. The subprime mortgage crisis and the subsequent bankruptcy of the Lehman brothers' that transacted deeply in the CDS market was proof of great risk when CDS is not properly controlled.

Mathematics and specifically calculus is widely employed in the analysis and dealing with derivatives. Derivatives are employed to portray the tendencies of monetary values and to predict their modifications in a specific period and within conditions. In the context of CDS, calculus is useful to estimate and express in numbers the risk and the value of these contracts through tools for modeling changes in credit spreads, default probabilities and so forth. Understanding calculus is very central when it comes to assessing financial models especially when Calculus BC involves both differential and integral calculus. Differential equations are used to model the way in which a variable, including the price of a CDS, changes from time to time. Whereas integrals are used to find the cumulative impacts that such changes will have such as the life expected exposure to be paid through a CDS contract. Hence, the employment of calculus in finance is not restricted to Credit Default Swap. It also applied in pricing of options, bonds, other types of derivatives, risk management, portfolio selection and to study market characteristics. To this end, through calculus, the financial analysts can derive models that will explain the trends of the financial assets under different conditions that make them foreplay better decisions in the management enterprises and reduce the impacts of worst docket scenarios [1-3].

Credit Default Swap is the financial contract through which the holder of the Credit Default Swap receives the credit risk referring to an underlying asset – corporate or sovereign – from another party. As for the CDS, the buyer aims to be insured against the risk of default and, at the same time, the seller takes this risk in exchange for receiving regular premiums. CDS contract comprises other features such as the notional amount, the reference entity, the credit events and the amount and frequency of payments referred to as the premium. The notional amount describes how much of the debt is insured and the figures can range from tens of millions of dollars to billions. This amount is set but never tendered between the two parties and instead, it is used to arrive at the premium rates and appropriation of claim costs. The reference entity is the borrower on whose credit the security is being bought or sold to hedge or gamble on its creditworthiness. Credit events under which the protection seller pays the protection buyer under the CDS contract are default, bankruptcy, failure to pay and restructuring. These are paid by the protection buyer to the seller and are commonly known as CDS spread and are usually paid on a quarterly basis. Such payments are expressed as a proportion of the notional and vary according to the credit risk assessment of the reference counterpart. Where a credit event takes place, the seller provides the buyer with payment of the difference between the notional amount and the market value of the defaulted obligation through which the buyer is made whole. CDS contracts are traded, over the Counter (OTC), which implies that they are traded directly between the buyer and the seller and not through a market. This over-the-counter characteristic of CDS contracts is that the seller has the legal right to refuse to buy back the protection or fulfill his obligation if a credit event happens. To manage this risk, the execution of CDS contracts has moved to central counterparties (CCP) who become the counterparty to each party to the contract.

CDS pricing is a difficult process, where credit risk of the reference entity must be evaluated, and proper premium payments have to be decided. There are several determinants of CDS price: credit spread of the reference entity, risk free interest rate, expected recovery rate and the market's expected default rate. Credit spread as a payment made by the buyer to the seller can be analyzed for the pricing of CDS. This spread is referred to as the default probability, which is the market's perception of the probability of an organization or company being unable to honor its debts. The spread is most often stated in basis points (one basis point equals 1/100 of one percent). 01%. For instance, on a CDS spread of 100 bps on a notional amount of $10 million, the annual premium was $100,000.

The estimation of the default probability of the reference entity is a key in pricing a CDS [3-6]. This probability can be estimated in many ways which include the hazard rate model which shows the rate of defaults over time. The hazard rate is dependent on time and is the rate of credit risk that is associated with the reference entity. The hazard rate λ(t) is employed in the modeling of the probability of default at time t The other parameter in CDS pricing is the recovery rate which represents the proportion of the notional amount which will be paid to the buyer if the reference entity defaults. The recovery rate R is usually an estimate arrived at from experience and realistic market value to be expected in the event of issued debts' default. The CDS spread is inversely related to the recovery rate: a lower expected recovery rate means that a higher spread of CDS will prevail as doing otherwise would put the risk on the seller. Another key parameter in the valuing the CDS is the risk-free interest rate r. This rate is the rate of returns on the investments with no risk, such as government securities and it is used in the discounting process of the future cash flow indicated in the CDS contract. The monetary amount which is expected to be lost due to default can be obtained by discounting the loss at the rate of risk-free rate. The cost of a CDS can be given by the future stream of premium payments combined with the expected loss in the event of a default. Mathematically, the CDS spread S can be derived by equating the present value of the premium payments to the present value of the expected loss: Mathematically, the CDS spread S can be derived by equating the present value of the premium payments to the present value of the expected loss in the equation (1):

$$S \cdot \sum_{i=1}^{n} \frac{1}{(1+r)^{t_i}} = (1-\mathcal{R}) \cdot \sum_{i=1}^{n} \lambda(t_i) \cdot \frac{1}{(1+r)^{t_i}} \qquad (1)$$

· S is the CDS spread

· ti is the time of the ii-th premium payment.

· λ(ti) is the hazard rate at time titi

· R is the recovery rate

· r is the risk-free interest rate

This equation at the bottom is used to describe the dependence between the CDS spread, the hazard rate, and the recovery rate. Thus, one states that by resolving this equation it is possible to define the CDS spread under condition of estimated default probabilities and recovery rate.

The CDS market has evolved and grown very big and is now considered to be among the largest and most active markets in the world. This has been occasioned by the need to address the issue of credit risk management and the use of CDS in speculative business. Nevertheless, the opportunities for economic growth that began with the development of the CDS market contain certain risks and problems. Counterparty risk would be one of the biggest dangers of CDS: it occurs when one of the participants of the contract defaults. This risk was clearly illustrated when the credit crisis of 2008 occurred and major banks such as Lehman brothers and AIG collapsed, sinking the market for CDS contracts. CDS contracts made financial institutions linkages in such a way that failure of one triggered failure in the other [7-10]. Market manipulation and pushing of speculative bubbles are the other risks associated with CDS. The existence of CDS is somewhat 'optional' since they provide the means for investors to speculate on the credit risk of entities without holding the actual bonds, thus bringing down the price of bonds and raising the cost of credit for corporations and governments. This speculative use of CDS can therefore fuel volatility in the credit markets, and cause instability and rise in cost of borrowing to issuers. Owing to the systemic risk that has been associated with CDS, authorities have taken measures intended at improving disclosure and alleviating counterparty risks. One of such measures is the central counterparties for CDS contracts whereby the contract is traded through a central counterparty hence eliminating the counterparty risk. Also, the authorities have prescribed CDS reporting requirements and capital requirements for institutions which have high levels of exposure to CDS to ensure that those institutions have enough capital to absorb possible losses. However, there are several risks associated with CDS that have been discussed above, yet CDS are still helpful in managing credit risk in the market. When employed efficiently CDS help in creating efficiency and stability in the market by permitting institutions to manage the credit risk and optimize the use of capital. However, this raises many concerns over the use of the technology and the impact of its misuse, and this calls for strong risk management measures and regulatory interventions.

2. The Mechanics and Functionality of CDS

2.1 Calculus and Mathematical Models for CDS Pricing

Calculus is important in financial modeling in that it enables the formulation of the mathematical model used to describe the behavior of financial assets. Calculus forms the basis of modeling the dynamics of credit spreads, default probabilities and other variables which determine the value of CDS contracts. Differential equations which are used to express the rate of change of a variable with respect to time are most effective in the valuation of CDS. From the above equations, one can establish how the price of a CDS varies in the face of change in market factors like interest rates and credit spreads. While derivatives are employed in determining the instantaneous variation of such factors, integrals are applied in determining the total impact of these variations over the life of a CDS contract, for instance, the total expected losses. In finance calculus with probability theory and statistics is applied to design models that can forecast how financial products will behave under certain circumstances. These models are very important in valuing derivatives, measuring risks and in effective decision making for investment purposes. When it comes to CDS, calculus-based models enable one to put a price on the risk of default and hence the value of the contract, which in turn helps to understand the credit markets. Of great importance in the mathematical analysis of CDS is the differential equation that defines the price of a CDS at a given time t. This model incorporates the basic parameters that determine the price of CDS such as the risk-free rate, the default intensity (hazard rate) and the recovery rate. The differential equation for CDS pricing can be expressed as follow equation (2):

$$\frac{\left(dP(t)\right)}{dt} = rP(t) - \lambda\left(1-\mathcal{R}\right) \quad (2)$$

· P(t) is the price of the CDS at time t
· r is the risk-free interest rate
· λ is the hazard rate, representing the default probability
· R is the recovery rate

This differential equation gives the dynamics of the CDS price with respect to time, including the interest rate on the notional amount and the expected loss on default. The symbol rP(t) denotes the interest on the CDS price, while the symbol λ(1−R) denotes the expected loss on account of default. The above obtained differential equation gives the price of CDS at any given time in function of initial conditions and parameters. This equation can be solved to find out the way through which the price of CDS is affected by changes in risk free interest rate, hazard rate and the recovery rate. This information is crucial for the valuation of the CDS contracts and for evaluating the risk of these products. To solve the differential equation, it is possible to apply the standard techniques of calculus, for instance the separation of variables or the integrating factor. It will depend on the shape of the hazard rate λ(t) and the recovery rate R, which are usually represented as a function of time to incorporate the dynamics of the credit risk of the reference entity. For example, if the hazard rate λ(t) is assumed to be constant over time, the differential equation can be solved analytically to obtain the following expression for the price of the CDS:For example, if the hazard rate λ(t) is assumed to be constant over time, the differential equation can be solved analytically to obtain the following expression for the price of the CDS in the equation (3):

$$P(t) = P(0) \bullet e^{(r-\lambda(1-\mathcal{R}))t} \tag{3}$$

where P (0) is the initial price of CDS. This solution shows that the price of the CDS rises at a very fast rate over time if the risk-free rate of interest 'r' is higher than the expected loss from default λ(1−R). On the other hand, if the expected loss is higher than the interest rate then the price of the CDS will gradually come down. In the more complicated models where the hazard rate as well as the recovery rate are functions of time, then inevitably one must use numerical methods to solve the differential equation. These methods involve the combination of time discretization where the equation is solved at each time step using techniques such as finite difference methods or Monte Carlo simulation.

Calculating the expected loss due to default is one of the most important aspects of CDS pricing, wherein integrals are of great use. The expected loss L can be expressed as the present value of the future losses, discounted at the risk-free interest rate: The expected loss L can be expressed as the present value of the future losses, discounted at the risk-free interest rate in the equation (4):

$$L = \int_0^T \lambda(t) \cdot (1-\mathcal{R}) \cdot e^{-\tau t} dt \tag{4}$$

· T is the maturity of the CDS
· λ(t) is the default probability at time tt
· r is the risk-free interest rate
· R is the recovery rate

This integral determines the present value of the expected loss blurred with the time factor known as the time value of money. This is summed over the life of the contract to give the total expected loss which is another important input into the CDS spread. This is especially so where the default probability λ(t) is not constant but is a function of time t. For instance, if the default probability rises whenever the credit standing of the reference entity is poor, the integral will reflect the rising default risk as well as its implication on the CDS pricing. For a given function λ(t), the integral might be calculated numerically, for instance, with the help of the trapezoidal rule or Simpson's rule. These are numerical methods which estimate the value of the integral by splitting the time axis into several segments and then adding up the contributions from each segment. In particular, the integral for expected loss is employed in combination with the differential equation for CDS pricing in practice. Thus, in this paper we propose the use of the CDS spread and the credit spread as two mathematical models for valuing CDS contracts and estimating the risk involved.

As examples of the concepts discussed above, let us consider an example of a financial organization that employs CDS for isolating credit risks connected with a corporate borrower. Let the bank have a large exposure to a corporation for which there exist signs of decreasing credit worthiness and potential default. The bank uses a CDS to hedge on the possibility of the corporation defaulting and, in effect, it sells the CDS to protect itself. By substituting the values of the different variables into the differential equation of CDS pricing the bank can determine the fair value of the CDS contract in the market. Another interesting characteristic of the model is the dependence of the hazard rate λ(t), which, as has already been mentioned, reflects the default probability, on time to distinguish the increasing risk. The recovery rate, denoted by R is influenced by past data and type of the corporate debt. In this case the price of the CDS and its future behavior is predicted and used by the bank as an outcome of the differential equation. The bank also needs to determine the expected loss, and for this is uses the integral for expected loss, which gives the approximate totals of the loss in the life cycle of the CDS contract. The above information helps the bank to know the right cures spread of credit and other related risks hence making the right decisions in risk management of institutional value. For instance, where the bank considers that the probability of a default of the corporation is going to rise in the following years, it can have two alternatives: to buy more CDS or to raise the premiums to be paid for the insurance. The decisions made are quantified by this model and thus ensures that the bank minimizes credit risk.

2.2 CDS and Market Stability

Credit Default Swaps thus for and against the stability of the financial markets. On the credit risk side CDS act as a means by which credit risk may be reduced and an opportunity through which default risks can be hedged. This may also lead to improvement of the market stability due to diminished potential for significant losses for the individual institutions alongside improved capital allocations. Still, the increased reliance on CDS is accompanied by systemic risk because various market participants rely on these instruments too much. When the financial crisis began in the autumn of 2008, the CDS unleashed significant social instability on the financial structure. Lehman Brothers and AIG, two of the biggest players in the CDS market, went bankrupt and the repercussions which swept over the rest of the financial market. There are therefore two big problems with CDS, that is the fact that markets of CDS can be manipulated, and that one can get into speculative bubbles. Known as credit derivatives, since CDSs let people make a wager on default risk independent of outright ownership of bonds, they can be applied to place downward pressure on the price of the instrument and upward pressure on cost for borrowing for organizations and regulatory authorities. This speculative use of CDS can therefore build up synthetic credit risks to turbulence within the credit markets and hence higher financing costs to issuers. Due to the systemic risk that CDS has, regulators have been forced to put measures that seek to enhance the transparency and cut short counterparty risk. These measures include A central counterparty will clear CDS contracts or in simple terms, there will be legal obligation or a central body to assure the proper performance of the contract. Moreover, reporting rules and existing and new capital requirements have been established for institutions that engage in CDS to largely guarantee their ability to meet losses. However, these risks do not diminish the importance of CDS in financial markets because it offers a way to hedge on credit risks. In this context, CDS can improve market efficiency as well as stability by enabling such institutions to manage their credit risk and channel their capital more efficiently. Nevertheless, the opportunities to exploit and the related risks reveal the necessity to develop reliable measures of risk control and legislation. The measures that have been taken up by the regulators in a bid to respond to the aspect of CDS and other financial derivatives following the financial crunch in 2008 cannot be underestimated. Among these measures, the more important one is trading and margin requirements in CDS contracts that imply that such contracts must be cleared through CCPs. CCP flows in between buyers and sellers and issues guarantees to counterparty risk of the respective contracts. Centralization also enhances the transparency of the CDS market and assists the regulators to have information concerning the CDS transactions. This makes it possible for the regulators to if you know observe buildup of risk in the financial system and come in and do something about systemic risk. For example, the Dodd-Frank Wall Street Reform and Consumer Protection Act in the United States required that the clearing and reporting of CDS contracts must be centralized and that institutions that trade in CDS must obtain higher capital and margin requirements. Apart from central clearing, license authorities have put other measures in place to control the risk of CDS. These are prohibition of short selling, increased capital charges for firms with large CDS exposures, and increased reporting on firms' CDS activities. Their functions are to assure that institutions could have necessary capital to compensate potential losses and to restrain the emergence of too subsequent risk within the economy. Algebraic equations like the differential equations and the integrals used in this paper are central to this process hence they are widely incorporated into regulatory systems. These models are used to evaluate the quantitative exposures on CDS contracts and to check and confirm the adequacy of capital to absorb those exposures. From assessing these structure calculus-based models, more information of CDS markets is uncovered to regulators and actions to reduce systemic risk can be taken. For instance, it is possible to apply these models to envision various situations for the CDS market: the rise of default frequency or the decrease of recovery rate. Regulators can therefore use the outcomes of such simulations in finding weaknesses in the financial structure and rectifying the same.

3. Conclusion

This paper has analyzed Credit Default Swaps, and in particular its features regarding the price quotations, risks, and on the financial markets. Thus, through the adoption of concepts from Calculus BC, we have endeavored to offer a pool of instrumentalities to examine and quantify CDS. The differenzial equation used for CDS pricing and the integral for expected loss are the equations that will help to describe the CDS dynamics and describe the interrelation between the parties and manage the financial risks. We have shown, therefore, how calculus-based price models can be employed for pricing CDS, measuring default risk as well as the influence of CDS on market stability. We have also stressed on the actual viability of appropriate risk management controls as well as monetary framework as crucial cornerstones of abolishing nuisances of CDS. Touching on the mentioned field, there is much more to be done in the future on the topic of CDS and financial mathematics. Nevertheless, the extended models, that involve counterparty risk, the liquidity, and the sentiment of markets, can disclose more profound information about the CDS markets. Also, the studies regarding legal consequences of CDS and the application of the mathematical modeling tools in risk management can contribute to the better understanding of the causes of crises on the market. Another research area could be the generalization of CDS data by building stochastic models in order to represent changes in CDS spreads over time. These models could include features like volatility, the ability to vary interest rates or creditworthiness that would give a better picture of risks tied to CDS. One of the areas of research is the application of machine learning and artificial intelligence in pricing and hedging of CDS. Applying such approaches as machine learning to the analysis of big data associated with CDS transactions, it would be possible to identify, understand and discover aspects that a mathematical model would not discern. This may cause more rational valuation of CDS and sounder technicus of risk Management in the financial market.

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