

Comprehensive Study and Practical Application of the Full Potential Equation in Addressing the Complexities of Transonic Flow Regimes

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Abstract:

This paper delves into the application of the full potential equation in understanding and addressing the complexities of transonic flow, a critical area in aerospace engineering and energy dynamics. By conducting a comprehensive literature review, the study explores various facets of the full potential equation, including numerical simulation, flow characteristic analysis, and implications for aircraft design. Although the full potential equation effectively captures certain physical aspects of transonic flow, challenges arise when applied to complex geometries and unsteady flows. Issues of computational efficiency and accuracy are persistent obstacles that limit its application. Despite these limitations, the full potential equation remains a cornerstone in the study of transonic flows, providing essential insights that lay the groundwork for theoretical advancements in the field. The paper emphasizes the importance of ongoing research and refinement of solution techniques, which are crucial for enhancing the effectiveness of the full potential equation. Improved methodologies will not only address the complexities inherent in transonic flows but also enhance the equation's performance in practical engineering scenarios, thereby advancing its utility in modern aerospace design and analysis.

Keywords: Full potential equation; transonic flow regimes; Full Potential.

1. Introduction

Transonic flow represents a significant area of study within fluid dynamics due to its complexity and the

critical role it plays in sectors such as aerospace and energy dynamics. As flow velocities approach the speed of sound, various phenomena such as shock waves and expansion waves complicate the flow

field, presenting profound implications for the design and performance of aircraft and other machinery. The full potential equation has emerged as an indispensable tool in this domain, offering vital insights into the behavior of transonic flows under diverse conditions.

Research Problem: Despite the advancements facilitated by the full potential equation, there remain considerable challenges in accurately predicting and managing the dynamics of transonic flow. These challenges include dealing with complex geometries, unsteady flows, and the inherent instabilities of transonic phenomena. Theoretical exploration and practical application are often hampered by limitations in computational efficiency and the precision of existing methods. These issues necessitate ongoing research to enhance the capabilities of numerical simulations and to refine the analytical approaches used in studying transonic flows.

This Paper's Contribution: This paper provides a comprehensive review of recent literature concerning the full potential equation, focusing on its application to transonic flow analysis. The study covers several key areas: numerical simulation techniques, flow characteristic analysis, and the practical implications of these studies in aircraft design and other related fields. By synthesizing findings from recent research, the paper seeks to offer a detailed examination of the theoretical foundations, numerical methods, and practical applications of the full potential equation. Additionally, it addresses ongoing challenges and highlights the need for further research to improve the understanding and application of this crucial equation in transonic flow. The ultimate goal is to advance both scientific knowledge and engineering practices by enhancing the precision and efficiency of solutions provided by the full potential equation.

2. Theoretical Foundations and Background

2.1 Definition and Development History of the Full Potential Equation

A key idea in fluid dynamics is the full potential equation. To put it simply, it's a mathematical expression for the flow of compressible and inviscid fluids.

The capacity of the whole potential equation to connect the fluid's velocity potential to different physical characteristics, such as pressure and density, is what defines it. It makes some assumptions, such as that the flow is rotationally free and viscous.

There are important turning points in the full potential equation's development history. It became a useful tool

for managing intricate fluid flow issues, particularly when compressible and high-speed flows were involved. The groundwork was established by early research, and as computing power increased, more sophisticated iterations and approaches were created.

2.2 Characteristics and Challenges of Transonic Flow

When a fluid flow condition approaches the speed of sound, it is referred to as transonic flow (Mach 1). Shock waves can arise in this domain when the flow speed suddenly shifts from subsonic (Mach < 1) to supersonic (Mach > 1) in concentrated locations. A shock wave develops in transonic flow as the fluid speed goes from subsonic to supersonic. A shock wave is a relatively small area when temperature, pressure, density, and other physical properties fluctuate dramatically. In contrast, an expansion wave occurs when the fluid speed drops from supersonic to subsonic. This is the reverse of a shock wave, causing the flow to slow down and the pressure to drop [1].

Shock waves in transonic flow induce abrupt temperature, pressure, and density changes that greatly increase aerodynamic drag and may even result in flow separation. The flow structure is extremely complex due to the presence of both supersonic and subsonic zones in the flow field. It is challenging to accurately mimic transonic flow because of this intricacy. For example, buffeting is a phenomena of flow instability in transonic flow brought on by flow separation and the interaction of shock waves and boundary layers. The phenomenon is frequent in transonic flow and has a negative effect on an aircraft's fatigue life and structural strength [2]. Shock waves can also cause drastic changes in parameters such as pressure, density, and temperature, having a significant impact on the performance and structure of aircraft.

When it comes to the transonic flow around complex geometries and unsteady transonic flow, the problem becomes more complex. In the transonic area, unstable air-flow phenomena like separation and vortices may occur, affecting aircraft stability and controllability derived from the HLFC wind tunnel study. It has been demonstrated that surface suction can impede the transition by altering the laminar base flow and suppressing the most unstable crossflow vortices. The dominant mode characteristics of the secondary instability stage will be impacted by the surface suction's substantial delay in the saturation area and weakening of the peak amplitude of the saturated crossflow vortices [3].

The complexity of transonic flow causes many difficulties in experimental verification. Designing and conducting experiments of transonic flow require high-precision

measurement equipment and complex experimental techniques. Moreover, the comparison and verification of experimental results and numerical simulation results also need careful analysis and processing. The early experimental and computational fluid dynamics research of moist air transonic flows demonstrated a notable shift in the flow field structure when atmospheric air relative humidity increased. The accuracy and resilience of the used in-house CFD code for the transonic flows with weak shock waves are confirmed by comparisons of the CFD findings with experimental data for both dry air test cases [4].

2.3 Applicability of the Full Potential Equation in Transonic Flow

Assuming irrotational flow, the complete potential equation is used to analyze transonic flows, which reduces the complexity of the flow control equations and improves computational efficiency. Because it makes it possible to quickly calculate the fundamental features of the flow field, this method is very helpful for preliminary analysis. The approach provides preliminary estimates of the velocity field, identifies broad flow trends, and supports design and optimization efforts. It performs well in low-speed to low-speed supersonic situations. Shock waves and flow separation are two complicated phenomena that arise when the flow speed approaches the speed of sound because of the large changes in fluid density and pressure that take place. These effects are difficult for the full potential equation to adequately predict, particularly in shock wave zones where abrupt discontinuities in fluid characteristics occur. As a result, although the equation can provide approximate values for the locations of shock waves, these values are frequently inaccurate. More complex computational techniques or experimental validation are usually needed for accurate simulation of shock wave power and position. Furthermore, flow separation requires complex boundary layer behavior that is beyond the modeling capabilities of the full potential equation. More sophisticated computational fluid dynamics (CFD) techniques and empirical data are typically required to adequately handle these complex processes.

3. System Analysis and Application Research

3.1 Numerical Methods and Discretization Techniques

The problem domain is divided into discrete elements by the Finite Element Method (FEM), which uses shape func-

tions to approximate solutions within each element. FEM handles complex geometries and boundary conditions well by combining local solutions into a global solution. This approach is especially helpful for structural analysis and aerodynamic design in transonic flows, where complex forms and boundary conditions are typical. FEM is an effective instrument for researching and applying solutions to the complicated problems of transonic flows because of its precision and flexibility. This method for solving the full potential equation within a finite element framework. This approach employs a background mesh and a continuous level set function, ϕ , to accurately capture complex geometries and manage high Reynolds number flows. It is also well-suited for complex aerodynamic applications, including commercial aircraft [5].

The Finite Difference Method (FDM) solves numerical problems by converting partial differential equations (PDEs) into algebraic equations. This is done by dividing the computational domain into a regular grid and using finite differences to approximate derivatives at the grid points. For example, second-order derivatives are approximated by differences between the current point and its neighbors. While FDM is simple and straightforward to implement, it can struggle with complex geometries and boundaries. It may also encounter numerical instability with high gradients or discontinuities, often requiring higher-order difference schemes and stability checks to ensure accuracy discretizes the Euler and Navier-Stokes equations, which describe fluid dynamics under transonic conditions, into algebraic equations and solves them on a structured grid. The focus is on refining the grid in high-gradient regions, such as near shock waves, to enhance accuracy [6].

The Finite Volume Method (FVM) tackles PDEs by breaking the computational domain into multiple control volumes and applying conservation laws within each volume. This approach translates global conservation principles into local algebraic equations, maintaining conservation properties. FVM is well-suited for handling complex geometries and problems with strong flow gradients, such as shock waves and boundary layers. It is particularly effective in fluid dynamics applications, where it can manage irregular grids and boundaries while providing high accuracy and stability in computations. FVM divides the computational domain into several control volumes and applies conservation laws to each of these volumes. By translating global conservation principles into local algebraic equations, it ensures that conservation properties are preserved. This approach is particularly effective for managing irregular grids and boundaries, and it excels at accurately capturing complex flow features such as shock waves and boundary layers [7].

By tackling the problems of transonic flow, these discretization and numerical approaches offer strong tools that improve model correctness and computational efficiency.

3.2 Case Analysis of Transonic Flow Scenarios

Transonic flow is a situation where a fluid approaches or crosses the speed of sound (about 343 m / s) during flight or flow. Given that contemporary aircraft, especially those used in warfare, are designed to be extremely fast, incredibly agile, and lightweight. Although these design objectives improve performance, they also expose the aircraft to a number of transonic aeroelastic problems. Flutter, divergent bending, and twist are examples of phenomena that can occur at transonic speeds as a result of the interplay between structural flexibility and aerodynamic forces, thereby jeopardizing stability and control. Maintaining structural integrity and performance at these speeds presents difficult problems due to the combination of high speed and aerodynamic forces. Besides, apart from the typical bending-torsion flutter, transonic aeroelasticity presents other distinct occurrences. The swept-back wing undergoes a decline of the flutter boundary in the transonic regime, which is known as the transonic dip phenomenon [8]. First, the program optimizes the wing's overall aerodynamic shape globally, taking into account the wing's general structure, aerodynamic properties, and fuel consumption implications. Subsequently, the local design is improved by precisely modifying particular wing elements, including the airfoil's curvature, the leading and trailing edges' designs, and the internal structure's arrangement. By focusing on particular aerodynamic loads and structural needs, this meticulous adjustment procedure further optimizes wing performance and ensures that fuel consumption during transonic cruise is kept to a minimum while retaining structural strength and stability [9]. As a result, transonic aeroelasticity continues to be a persistent challenge in the aerospace field.

3.3 Practical Application of the Full Potential Equation in Engineering Fields

Aerodynamic performance of aircraft is intimately related to fluid mechanics; consequently, in order to execute optimization and design tasks, it is critical to accurately describe and anticipate the features of the flow surrounding the aircraft. Boundary layer: A thin layer forms quite close to the airplane surface. This region is crucial to the design of airplanes, but modeling it can be very difficult, especially when the boundary layer is turbulent (high Reynolds numbers) [10]. Optimizing aerodynamic performance in aerospace engineering requires a precise simulation of the complex and variable airflow surrounding aircraft.

Engineers can obtain an essential theoretical basis for optimizing wing designs and the overall shape of aircraft by employing the full potential equation. By reducing air resistance, this simulation helps to increase lift and stabilize the aircraft. Moreover, it assists in optimizing maneuverability and stability, resulting in a significant enhancement of overall flight performance. By means of these simulations, in-depth analysis enables focused modifications to design parameters, guaranteeing the optimization of both aerodynamic efficiency and operational effectiveness.

In the field of energy engineering, the full potential equation helps engineers simulate the flow of air through various components of a gas turbine, including the compressor, combustion chamber, and turbine. By solving the full potential equation, engineers can obtain the distribution of air velocity, pressure, and temperature. Using the flow data generated by the full potential equation, they can optimize the geometry of the turbine blades and the configuration of the channels. Precise simulation of airflow allows for adjustments to blade angles and channel layouts, reducing flow resistance and maximizing aerodynamic efficiency, thereby improving the overall performance of the gas turbine. Based on the flow simulation results from the full potential equation, engineers can make design improvements to address potential flow issues, such as flow separation and shock effects. These modifications help enhance the stability and reliability of the gas turbine [11]. In the aspect of automotive engineering, the full potential equation can conduct precise design and optimization of the vehicle shape. By in-depth analysis of the interaction between the vehicle and the air during driving, it can effectively reduce the drag coefficient. Recently, design engineers are applying aerodynamic principles to boost vehicle efficiency. Since aerodynamic drag consumes about half of a vehicle's energy, research emphasizes using the full potential equation in flow simulations and design optimization to minimize drag and enhance fuel economy [12]. In the field of water conservancy engineering, for the water flow conditions in dams, channels and pipelines, the full potential equation can conduct comprehensive and accurate analysis. This is of great significance for optimizing engineering design, reducing energy loss and preventing unstable water flow problems, and can ensure the safe and stable operation of water conservancy facilities and improve the utilization efficiency of water resources.

4. Challenges and Limitations

Modern aviation engines are not complete without transonic axial flow compressors because they are vital to the optimization of the pressure ratios at each stage. These compressors do, however, have several restrictions and

difficulties. Reduced efficiency and stability may result from complicated interactions, such as shock waves and flow separation, that the compressor's airflow may encounter at transonic speeds. To guarantee dependable performance under a variety of operating situations, handling these problems calls for sophisticated design and simulation methodologies. Compressor designs are further stressed by the growing need for greater efficiency and power density, necessitating ongoing innovation to meet these technological obstacles. Significant research, both numerical and experimental, is dedicated to enhancing efficiency and stall margin, particularly at peak performance and near-stall conditions.

In practical engineering applications, the simplified model of the full potential equation may not fully capture the complexities of real aerodynamic environments, such as three-dimensional flow and turbulence effects. Given that the full potential equation assumes inviscid and irrotational flow, it may fail to accurately represent various phenomena present in actual flows, leading to discrepancies between simulations and real conditions. Therefore, engineers often combine the full potential equation with other computational fluid dynamics (CFD) methods and experimental data to address its limitations. This integrated approach enhances the accuracy of predictions and improves design precision and reliability, ensuring that optimized designs meet practical engineering requirements under complex aerodynamic conditions.

5. Conclusion

This study has underscored the indispensable role of the full potential equation in understanding and analyzing transonic flows, highlighting its necessity for elucidating the complexities of flow fields where subsonic and supersonic regions interact. While the equation provides a robust theoretical foundation for exploring transonic phenomena, it faces challenges when applied to complex geometries and erratic flows, signaling a need for advancements in accuracy and computational efficiency. The insights derived from this research have profoundly impacted the aerospace and energy sectors, informing the development of aircraft and turbomachinery by providing essential theoretical and practical knowledge.

It is imperative that future research focuses on developing more sophisticated numerical methods and computational strategies to more effectively address the complexities associated with transonic flows. Enhancing the accuracy of these models through experimental validation will be critical, ensuring that theoretical predictions align closely with real-world behaviors. Moreover, exploring the ap-

plication of the full potential equation in emerging areas such as advanced propulsion systems and next-generation energy technologies offers promising avenues for research and innovation. Collaborative efforts between academia and industry are essential to leverage the full potential of this equation, continuing its contribution to the advancement of transonic flow understanding and the evolution of related technologies. By tackling these challenges and exploring new applications, the ongoing research will not only refine current methodologies but also expand the scope of transonic flow studies to include a broader range of engineering problems.

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