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Interplay of Markov Chain and Random Walk in Several Areas

Keshu Qian

College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, China

Corresponding author: qiankeshu@nuaa.edu.cn

Abstract:

The research paper discusses the significance of Markov chain and random walk as fundamental probabilistic models that describe transitions between states in various fields, including natural language processing, finance, and bioinformatics. It traces the historical development of random walk, beginning with Karl Pearson's initial concept in 1905 and culminating in the emergence of quantum random walks in the 21st century, while highlighting their mathematical properties and applications. The paper categorizes states in Markov chain into recurrent and transient, providing criteria for their classification and demonstrating the recurrence of random walks in different dimensions. It explores diverse applications of random walk, such as knowledge representation learning, Markov Decision Processes in reinforcement learning, advancements in medical research through neuroimaging, and innovative strategies for waste recycling path planning. The conclusion emphasizes the ongoing evolution of random walk theories, particularly with advancements in quantum computing and big data, suggesting that their applications will continue to expand and become increasingly sophisticated in the future.

Keywords: Markov chain; Random walk; Markov Decision Processes.

1. Introduction

The Markov chain is a robust probabilistic model that effectively delineates the transitions of random systems among a finite set of states. Its characteristic of "memorylessness" not only facilitates the modeling of intricate random processes but also establishes the Markov chain as an essential analytical instrument across diverse domains, such as the modeling of linguistic structures in natural language processing, the forecasting of price variations in financial markets, and the examination of genetic sequences in bioinformatics. The random walk, a quintessential illustration of a Markov chain, further underscores this property of memorylessness. It represents a straightforward and intuitive random process wherein a walker (or particle) moves randomly within a defined space, with the direction and distance of each step determined exclusively by the current position, independent of prior trajectories. This model serves as an abstract generalization of numerous diffusion phenomena observed in nature, including the Brownian motion of gas molecules and the diffusion of solutes in liquids, thereby providing a robust theoretical framework for the investigation of these phenomena. In conclusion, both the Markov chain and its specific instance—the random walk—significantly enhance people's comprehension of random phenomena and occupy an irreplaceable position across multiple disciplines.

The exploration of random walks can be traced back over a century, with Karl Pearson first introducing the concept in 1905 as a model to describe and analyze the statistical characteristics of random activities [1], analogous to the erratic movements of an individual under the influence of alcohol. Initial research predominantly concentrated on the mathematical properties and theoretical underpinnings of random walks, alongside their applications in computer science. Noteworthy applications include PageRank [2], computer vision [3], and the analysis of complex social networks [4]. In the early 21st century, advancements in quantum computing technology led to the emergence of quantum random walks as an extension of classical random walks within the quantum mechanics framework, gradually becoming a focal point of scholarly inquiry. Lov K. Szegedy developed a quantum variant of Markov chains [5], which facilitated the formulation of novel quantum walk algorithms. Magniez and colleagues further expanded the applicability of Szegedy's methodology to a broader class of ergodic Markov chains, thereby enhancing previous complexity results based on the interrelations among the eigenvalues or singular values of the corresponding Markov chains [6].

This article commences with an examination of the recurrence properties of the random walk model, demonstrating the conditions under which random walks exhibit recurrence, and subsequently extends to the applications of random walks across various fields.

2. Theory of Markov chain

2.1 Recurring Characteristics Summary

In a Markov chain, there are two primary categories of states: recurrent states and transient states. A recurrent state is characterized by the system's eventual return to that state after entering it, with a probability of 1. On the other hand, a transient state has a probability of returning to it that is less than 1. The distinction between recurrent and transient states is important as it greatly influences their long-term behavior. Thus, it is essential to correctly identify whether a state is recurrent or transient to effectively analyze and comprehend the long-term dynamics of Markov chains.

To effectively distinguish between recurrent and transient states in Markov chains, researchers have proposed various classification criteria. Here are some of the main criteria:

Let $P_{ij}(n)$ be the probability of first returning to state j after n steps starting from state i, and $G_{ij} = \sum P_{ij}(n)$. Based on properties of return probabilities, one can derive the following criterion:

(1) Determination of recurrent states: If $G_{ii} = \infty$, then state i is called recurrent. This means that the total probability of the system returning to state i after starting from state i is divergent, that is, the system will return to state i infinitely many times.

(2) Transient state determination: If $G_{ii} < \infty$, then state i is called transient. This means that the total probability of the system returning to state i after starting from state i is finite, which indicates that the system can only return to state i a limited number of times.

In a Markov chain, if state i and state j are communicating (there exist positive probability paths from i to j and from j to i), then they are either both recurrent states or both transient states.

The random walk problem, as a case of Markov chains, can also be proven to be recurrent using the methods mentioned above.

2.2 Recurrence of Random Walks in Different Situations

For one-dimensional symmetric random walk and two-dimensional symmetric random walk, one can prove that it is recurrent, while for the three-dimensional symmetric random walk, one can show that

$$P_{ii}(2n) = \binom{n}{k} \sum_{k,j>0}^{n} \sum_{k+j\leqslant n} \frac{1}{2^{2n}} \left(\frac{1}{3^n} \frac{n!}{k!j!(n-k-j)!} \right) 2 \leqslant k \frac{1}{n^{\frac{3}{2}}}$$

(where k is a constant). According to Green's formula, one knows that the three-dimensional symmetric random walk is transient.

For random walk problems in dimensions higher than three, one can first provide the relationship formula between dimension and probability

$$P_{ii}(2n) = \sum_{n=0}^{\infty} \frac{(2n)!}{(x_1!\ldots x_d!)^2} \frac{1}{(2d)^{2n}} (\sum x_d = n) k \frac{1}{n^{d/2}}$$ (where k is a constant). Therefore, for dimension d, $P_{ii}(2n)$ is a monotonically decreasing function, which means that for cases above three dimensions, the results are all very rare.

For the problem of asymmetric random walks, one can

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also start considering it from one dimension. In the case of one dimension, by replacing $1/2$ with pq, it is easy to get $P_{ii}(2n)\frac{(4pq)^n}{\sqrt{\pi n}}$. One can see that $G_{ii}=\infty$ if and only if $p=q=\frac{1}{2}$. Therefore, the one-dimensional asymmetric random walk is very recurrent.

In the two-dimensional case, substituting p,q,s,t for $\frac{1}{4}$ yields $P_{ii}(2n)\frac{1}{\pi n}\bullet\omega$ (if and only if $p=q=s=t$, $\omega=1$, and is independent of n. In other cases, $\omega<1$ and it is related to the size of n). Therefore, two-dimensional asymmetric random walks are very recurrent.

Random walks have a very special case known as the random walk problem with absorbing barriers. First, the term “absorbing barrier” means that once a particle reaches that position, it stops moving. A simple example can illustrate this. Consider a particle that moves one unit to the left or right at each time step, with a probability of α for moving left and a probability of β for moving right. The particle is absorbed at the positions $x=0$ and $x=a+b$, meaning that once the particle reaches either of these positions, it will no longer move. Now, the author wants to know the probability that if the particle starts at position $x=n$ (where $1\leqslant n\leqslant a+b-1$), it will eventually be absorbed at $x=a+b$. This probability can be realized in two ways: the particle moves to the right in the next step and is eventually absorbed at $x=a+b$; or the particle moves to the left in the next step but can still be absorbed at $x=a+b$ afterward. According to the law of total probability, one can sum the probabilities of these two scenarios to obtain the total probability of the particle being absorbed at $x=a+b$. The author denotes this probability as P_n, where n is the initial position of the particle. Therefore, for $n=1,2,...,a+b-1$, one has $P_n=\alpha P_{n+1}+\beta P_{n-1}$ [7].

3. Application of Markov chain

Random walks are not merely a theoretical mathematical concept; they also have numerous applications across various fields.

3.1 Knowledge representation learning

Knowledge representation learning is a method for transforming entities and relationships within knowledge graphs into low-dimensional yet information-rich vector forms. This approach contains profound semantic information that has direct applications in various tasks. Unfortunately, traditional knowledge representation learning models often focus solely on specific triples during the training process, neglecting the broader impact of other related entities and relationships within the knowledge graph on these triples. This limitation results in vector representations of entities and relationships that are relatively shallow on a semantic level, thereby diminishing their performance in practical applications to some extent. However, it is encouraging that the domestic research community has recently introduced a novel knowledge representation learning model. This model cleverly integrates advanced technologies, including random walk algorithms and Long Short-Term Memory (LSTM) neural networks. Compared to traditional models, this innovative approach places greater emphasis on the positional information of entities and relationship nodes within complex network structures, enabling it to capture and reflect the deep structural features of knowledge graphs more comprehensively [8].

Experimental data further confirms the model’s exceptional performance. The results indicate that the new model not only generates semantically rich and nuanced representation vectors but also achieves significant improvements in training efficiency. This accomplishment undoubtedly injects new vitality into the field of knowledge representation learning and equips people with more powerful tools for further exploration and application of knowledge graphs.

3.2 Markov Decision Processes

In addition to the field of knowledge representation learning, Markov Decision Processes (MDPs) represent a significant application scenario for random walks [9]. A Markov Decision Process (MDP) serves as a foundational mathematical framework frequently employed to model decision-making in stochastic environments. It comprises four essential components: the state space, which delineates the collection of potential states, each representing the current condition of the environment; the action space, which catalogs the array of actions available to the system in each state; the transition probabilities, which articulate the likelihood of the system transitioning from one state to another following the execution of a specific action; and the reward function, which specifies the immediate reward accrued by the system based on a given state and action. Within the MDP framework, the objective of the decision-maker is to ascertain a policy that prescribes the optimal action for each state, thereby maximizing the long-term cumulative reward.

Markov Decision Processes (MDPs) are extensively utilized within the domain of artificial intelligence, particularly in the context of reinforcement learning. Reinforcement learning algorithms are designed to determine optimal actions based on the current state of the environment, with the objective of maximizing rewards through ongoing interactions between the agent and its environment. This process involves the agent first observing the environmental state, subsequently executing an action, after which the environment responds by providing a reward contingent upon that action and transitioning to a new state. Through iterative cycles of this nature, the agent incrementally acquires the necessary strategies to effectively accomplish tasks. By leveraging Markov Decision Processes, reinforcement learning algorithms facilitate the agent's ability to devise optimal pathways in dynamic environments and to make decisions that are responsive to fluctuations within those environments.

3.3 Medical Field

Random walks have also led to significant advancements in the medical field, contributing to further research in mental illnesses and neuroscience.

With the continuous advancement of non-invasive neuroimaging techniques, such as diffusion tensor imaging and functional magnetic resonance imaging, researchers are gaining a deeper understanding of the brain's structure and function through the concept of networks. The application of these technologies has significantly propelled the field of brain science forward. The brain network, a complex system composed of hundreds of brain regions and their interconnections, plays a crucial role in elucidating the mechanisms underlying brain function. Currently, research on structural brain networks often emphasizes the direct connections between brain regions; however, the higher-order connections among these regions are also of considerable importance. Investigating these higher-order relationships enhances people's comprehension of the complexity of brain networks and offers new insights for the diagnosis and treatment of mental disorders [10].

The human brain is regarded as a highly dynamic and complex network, with its functional connectivity relying on structural connections to facilitate effective cognitive communication. The potential connections between two brain regions within the structural network provide significant insights into understanding functional communication. However, quantifying the higher-order relationships among different regions of the brain network remains a challenge. Although network embedding methods have been widely applied, they are typically designed for single networks and often fail to yield new insights. Nevertheless, by employing random walk methods based on one-step and two-step neighbor layers, it is possible to obtain embedding representations that are rich in information for each brain region. Utilizing these embedding representations allows for the capture of both direct and indirect connections within the network through the similarity of higher-order feature vectors between pairs of nodes. This approach shows promise for more comprehensively describing the relationships between brain regions in patients and elucidating abnormalities in brain function.

3.4 Waste Recycling

In the field of waste recycling, traditional methods that focus solely on the shortest routes between individual waste points and recycling stations may optimize locally but result in high global costs. These methods often overlook overall path efficiency, leading to increased total recycling expenses. As urban traffic networks become more complex and the types of recycling stations diversify, this issue has become increasingly pronounced, highlighting the need for an efficient waste recycling path planning system. However, attempting to find a globally optimal solution by enumerating all possible path combinations is impractical due to the exponential growth of computational demands as the network scales. Given that the volume of waste is typically comparable to the number of network nodes, the number of potential path combinations can reach a double-exponential level, further complicating the problem.

To address this challenge, Chinese scholars have innovatively introduced a random walk model and developed a straightforward yet effective strategy for garbage collection path planning [11]. This strategy begins at any node within the network and employs a random walk process to generate a series of random path chains of length l. The selection of l is critical, as it directly influences the accuracy and efficiency of the outcomes. Theoretically, as l approaches infinity, the model can approximate the optimal solution. By utilizing the random walk method, this strategy can swiftly yield near-optimal combinations of collection paths while requiring minimal computational resources and sampling times, thereby demonstrating high computational efficiency and practicality.

4. Conclusion

In theoretical research, the probabilistic assumptions, mathematical properties, and statistical laws of random walks have been extensively studied. For instance, the mean of the random walk model represents the deterministic component in random time series, while the variance

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indicates the extent to which the random component deviates from the mean. However, as research advances, traditional random walk theory encounters several challenges. Consequently, researchers have begun to investigate more complex and precise random walk models to better simulate the dynamic random phenomena observed in the real world. In the future, research on random walks is expected to evolve in a more diversified and in-depth manner. On one hand, with the continuous advancement of quantum computing technology, quantum random walk techniques are anticipated to be applied across various fields. Quantum random walks not only provide higher computational efficiency and enhanced security but also leverage properties such as quantum superposition and entanglement to create more intricate random walk patterns. This development will offer innovative solutions for optimization problems in sectors such as finance, healthcare, and energy. On the other hand, with the emergence of big data and artificial intelligence technologies, random walk models will increasingly integrate with these advancements to achieve more accurate predictions and informed decision-making. For example, by analyzing geographic location data related to human activities, more refined models of individual and group movement can be developed, thereby optimizing applications such as traffic forecasting, urban planning, and epidemic modeling. Simultaneously, these models can provide robust support for areas such as social network analysis and marketing. Reason: The revised text improves vocabulary, enhances readability and clarity, and corrects grammatical and punctuation errors while maintaining the original meaning. In summary, random walks, as a crucial model of stochastic processes, will continue to play a significant role in the future. With ongoing technological advancements and deeper research, the application areas of random walks will expand even further, and their theories and methodologies will continue to be refined and innovated.

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