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The Different Application of Definite Integral

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Abstract:

This article mainly introduces the application of definite integrals. There are four parts in this article, and each part corresponds to an example. The first part is calculation of definite integrals area of plane images. In the first part, two methods are used in one stop to provide different solutions to definite integrals in different questions. The second part is using define integration to calculate the volume of regular-shaped solid. This part explains the general formula used in childhood. The third part is definite integral to calculate the volume of a rotation solid, which explains how to use definite integrals to calculate the volume under the condition of only one coordinate system. The fourth part is using defined integration to solve kinematics problems., which describes the use of definite integrals in other subjects. Calculus not only provides precise mathematical descriptions and solving methods for physics, but also promotes the development and progress of various fields of physics.

Keywords: Calculation; application; motion.

1. Introduction

The definite integral

$$\int_a^b f(x)dx \tag{1}$$

represents the area between f(x) and the x-axis in the interval between a to b. But the area is not always positive. When f(x) lies below the x-axis within the interval between a to b, then the definite integral is negative. Additionally, when $\int_a^b f(x)dx$ is a certain value then $\int_b^a f(x)dx$ is the opposite of that value. However, there is another very important premise for the definite integral, which is that the function f(x) must be differentiable throughout the interval [a,b].

In 2015, Steven researched the definite integrals in pure mathematics and applied science contexts [1]. For the economic area, Zhen et al. addressed the design of sliding mode controller for an uncertain chaotic fractional order economic system [2]. Adrian et al. took a deeper sight of discretization for the integral equations in 2011, which could be a useful tool in engineering [3]. Definite integral is also applied in biology. Mehdi and Rezvan introduced the variational iteration and Adomian decomposition methods to solve the delay logistic equation [4]. Definite integral can also work on transmission problems. Martin and Stephan figured a direct boundary integral equation method for transmission problems [5]. Lai and his team used definite integration to improve the theorical models for well drilling in 2023 [6]. Zhang and colleagues used definite integration to set up a model to check the relationship between oil level and the rate of pushing the oil out of an oil tank which has an irregular shape in 2023 [7]. Ma and teammates used integration to research coal mine safety in 2024 [8]. Wei and teammates used integration to design the control system of cutting machines [9]. In 2020, a researcher used integration in the field of geologic survey [10].

2. Application of Calculation

2.1 Calculation for Definite Integrals Area of Plane Images

2.1.1 Geometric Interpretation

The definite integral calculates the net area which is the sum of positive area (above the x-axis) and negative area (below the x-axis) covered by the function curve.

Example 1: The graph of f is shown in Fig. 1 for $0 \leq x \leq 4$. What is the value of

$$\int_0^4 f(x)\,dx \tag{2}$$

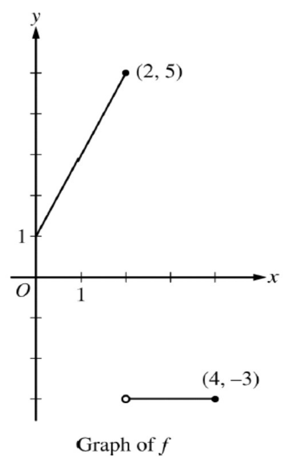

Fig 1: The graph of Example 1

Proof: Let f(x) = 2x+1 on the interval [0,2] and f(x) = −3 on the interval [2,4]. The purpose of this question is find the area between x-axis and f(x) in the interval [0,4].

To find the solution of this question, the integral is split into two parts:

$$\int_0^4 f(x)\,dx = \int_0^2 (2x+1) + \int_2^4 (-3)\,dx \tag{3}$$

Computing each part separately:

$$\int_0^2 (2x+1)\,\mathrm{dx} = 6 \tag{4}$$

And

$$\int_2^4 (-3)\,dx = -6 \tag{5}$$

Combining these two areas together gives the final answer.

2.1.2 Area Calculation of Specific Shapes

When an integral has large computational cost, it will be easier to calculate the area of specific shapes. For instance, if the area between x-axis and f(x) is a trapezoid, the formula of the trapezoid can be used on the question:

Area = (Upper Base + Lower Base) × Height/2

Example 2: Let f(x) = 2x + 1 on the interval [0,2] and f(x) = −3 on the interval [2,4]. The question is to find out the definite integral

$$\int_0^4 f(x)\,dx \tag{6}$$

Proof: Compute the area between x-axis and f(x) in the interval [0,4]. Similarly, the area needs to be split into two parts: the area between curve of f(x) on the interval 0 to 2 and the area between curve of f(x) on the interval 2 to 4. Computing these two parts:

$$\text{First Part Area} = ((1+5)*2)/2 \tag{7}$$

$$\text{Second Part Area} = 3 \times 2 = 6 \tag{8}$$

From this it's important to know that the area below the x-axis needs to be negative, so the second part area should be -6 instead of 6. Therefore, the final answer is 6 + (-6) = 0.

2.2 Using Define Integration to Calculate the Volume of Regular-shaped Solid

It's already known that the volume of a circular cone equals to one-third of the cylinder which has the same height and base area. They prove this with pouring water from the cone to the cylinder. But in this passage, the author will use integration to calculate the volume of a cone and then prove the conclusion.

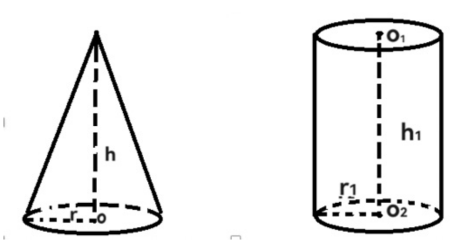

Fig. 2 Comparison cone of cylinder

In the diagram Fig. 2, assume that: h = h_1, r = r_1. So, the base area of the cone equals to that of the cylinder.

Proof: the volume of the cylinder is obviously:

$$v_{cylinder} = \pi r^2 \times h_1 \tag{9}$$

For the cylinder, it can be regarded as a lot of cylinders that have the same height stacked up with decreasing of radius along with the layer increases in Fig. 3.

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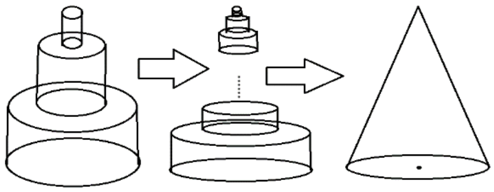

Fig. 3 Decomposing

Assume that $r=a$, the minimum of the cylinder in the layer approaches to 0, and the maximum approaches to a Then, the two boundaries can be confirmed as 0 and a .
The value of r increases linear with the increasing of the layer. So, the resultant height will be $\frac{h}{r}$.
The area of the circle $A=\pi r^2$ leads to the formula of the value of the cone:

$$v_{cone}=\frac{h}{r}\times\int_0^a \pi r^2 \tag{10}$$

$$\int_0^a \pi r^2 d_r=\frac{\pi a^3}{3}-\frac{\pi\times 0^3}{3}=\frac{\pi a^3}{3} \tag{11}$$

Thus,

$$v_{cone}=\frac{h}{r}\times\frac{\pi r^3}{3}=\frac{\pi r^2 h}{3}=\frac{1}{3}v_{cylinder} \tag{12}$$

Prove completed.

2.3 Definite Integral to Calculate the Volume of a Rotation Solid

The concept of calculating areas to find definite integral can be extended to find the volumes of solid of revolution. When the curve is around an axis so that the curve can be determined as a three dimensional shape.
Therefore, the goal is changed to find the volume of the shape by using integration.
The key idea of calculating the volume of a rotation solid is to cut the solid into infinite column, then plus all the volume together.
If the curve $y = f(x)$ is around the x-axis, the volume V of the solid can be calculated by the following formula.

$$V=\int_a^b\left[f(x)\right]^2 dx \tag{13}$$

Example 3. What is the volume of the solid generated when the region bounded by the graph of $x=\sqrt{y-2}$ and the lines x =0, y = 5 is revolved about the y-axis.
Proof: The region is bounded by x = y – 2 and the region of y=2 to y=5. Now using the disk method, the volume integral can be set up as:

$$V=\pi\int_2^5\left(\sqrt{y-2}\right)^2 dx \tag{14}$$

$$V=\pi\int_2^5\left(\sqrt{y-2}\right)^2 dx=14.137 \tag{15}$$

2.4 Using Defined Integration to Solve Kinematics Problems.

There have several simple methods or formulae to determine the displacement, acceleration and velocity of an object. When the acceleration keeps unchanged during the hole process of motion, the displacement can be easily found:

$$d=v_0\times t+\frac{at^2}{2} \tag{16}$$

However, if the object moves in variable acceleration, simple formulae can not be used to find the displacement anymore. In this passage, the author will use defined integration to determine the displacement of an object which moves in variable accelerates. how can the displacement be found without the formulae?

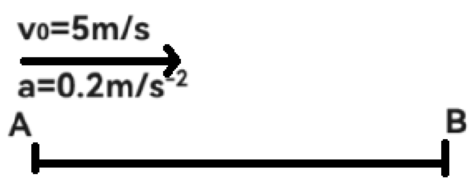

Fig. 4 Car motion

Example 4: A car travels from A to B in 5 seconds, with $v_0=5m\bullet s^{-1}$ and $a=0.2m\bullet s^{-2}$ as diagram Fig. 4 shows, how can we find the displacement of the car?
Proof: The vellocity increases linear as time t increases. Assume that $t=0$ when the car passes A. So the situation of the velocity of the car can be represented by a linear function:

$$v=f_{(t)}=5+0.2t \tag{17}$$

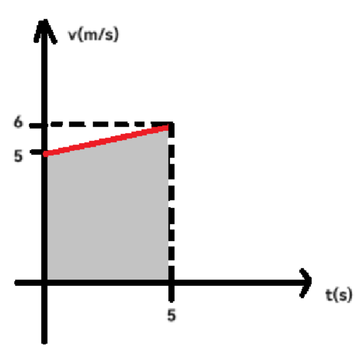

Fig 5. v-t diagram

A v£-t diagram can be drawn in Fig 5. The area of the shaded region is the displacement. From 2.1, the method of finding area by integration is known.

$$d = \int_0^5 5 + 0.2td_t = 27.5 \quad (18)$$

If acceleration linear increases while t increases, such as $a = 0.2t$, and v_0 remains the same, how can velocity be found?

According to $v_{instantaneous} = t \times a_{inatantaneous}$, velocity is the integration of acceleration.

$$v = \int 0.2td_t = 0.1t^3 + c \quad (19)$$

When t = 0, v_0 = 5, thus $5 = 0.1 \times 0^3 + c$, $c = 5$.

$$v = 0.1t^3 + 5 \quad (20)$$

Then

$$d = \int_0^5 \left(0.1t^3 + 5\right) d_t = \frac{325}{8} \quad (21)$$

In summary, no matter how the acceleration changes, the displacement can be found by calculating the area of the region bounded by t_0, t_v, X-axis and the line of the v – t diagram as well as the expression of v can be found.

3. Conclusion

To conclude, the article uses definite integration to calculate volume of substances in complicated shape, the area of irregular shapes, the distance traveled by cars with irregular acceleration. The article also gives examples of using definite integration to solve the problems in academic and people's life. During the research, the author found that it is important to use differential element method to analyze each process of calculation and it can be confirmed that the method of integration is the further application of differential element. The difference element method always plays important roles in academic research, such as physics, biology, chemistry, and archeology. The method can help people solve their problems in their daily lives. The thoughts of differential elements can be also used in people's works such as financial analysis, constructions industries. Thus, definite integration can provide some brand-new thoughts for solving problems, and this kind of skill can be easily learned by people so people can use it anytime if it is necessary. In the future research, the author will make more use of the thought of differential element so that the author can have better understanding of definite integration to do some further research. By improving the author's ability to solve some more difficult problems, the future researchers would discuss some complicated subjects such as improper integration, differential equations, multiple integration.

Authors Contribution

All the authors contributed equally and their names were listed in alphabetical order.

References

[1] Jones S. R. Areas, anti-derivatives, and adding up pieces: Definite integrals in pure mathematics and applied science contexts. The Journal of Mathematical Behavior , 2015, 38: 9–28.

[2] Wang Z., Huang X., Shen, H. Control of an uncertain fractional order economic system via adaptive sliding mode. Neurocomputing, 2012, 83:83–88.

[3] Lange A. W., Herbert J. M. Symmetric versus asymmetric discretization of the integral equations in polarizable continuum solvation models. Chemical Physics Letters, 2011, 509(1):77–87.

[4] Dehghan M., Salehi R. Solution of a nonlinear time-delay model in biology via semianalytical approaches. Computer Physics Communications, 2010, 181(7):1255-1265.

[5] Costabel M., Stephan E. A direct boundary integral equation method for transmission problems. Journal of mathematical analysis and applications, 1985,106(2):367-413.

[6] Lai J., Lu G., Zhou C. Calculation of integration in the model of tracks in the wells. Petroleum Drilling Techniques. 2023, 51(3):45-50.

[7] Zhang W., Yao K., He Y. The models of oil level in the oil tank depends on the algebra of fighting the fire. Computer Application and Software. 2023, 40(7):300-304.

[8] Zhang X., Li D., Ma Q. Research of coal and methane gas depends on boundaries of gaining energy. Coal mine safety. Mechanical Design and Manufacturing. 2023, 54(9):23-29.

[9] Zhang M., Wei W. Designation of the operating system in multiply- operated cutters. 2023(6):152-156.

[10] Yi M. The use of Partial least square in the geography. Microprocessor, 2020, 41(1):52-56.