Dean&Francis

ISSN 2959-6157

From infinitesimal to calculus: Leibniz and Newton's different perspectives

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Abstract:

Calculus is crucial for the foundation of mathematics and other science, especially in analysis, it creates a mathematical basement. It is invented in the 19th century, and many mathematicians from the past and present have proposed numerous calculus approaches. The most outstanding two calculus methods are Newton's fluxional calculus and Leibniz's differentiation and integration, which are widely used nowadays. Newton's method is mainly used in Physics and Leibniz's method contributes a lot to mathematical analysis. Though the study of Leibniz and Newton built the foundation of modern calculus, their views on calculus differed. Furthermore, infinitesimality has been essential to the development of calculus, and it is crucial for the research by Leibniz, while introducing Leibniz's method, it will be mentioned and explained. This paper will introduce Leibniz's and Newton's methods briefly and compare their difference. Leibniz and Newton both used infinitely small quantities to explain the constantly changing rate. Leibniz introduced infinitesimals to solve the problem of derivatives and integrals, while Newton used the term "flow number" to describe the process of change. In addition to simplifying calculation, the introduction of infinitesimal paved the way for the subsequently developed more rigorous limit theory.

Keywords: differentiation, calculus, infinitesimally, fluxional calculus.

1. Introduction

For at least 250 years, mathematics has been an essential component of human intellectual history and education. Researchers have not reached a consensus on the nature of mathematics, and a generally accepted definition has yet to be established, despite extensive investigation over a prolonged period [1].

Calculus is one of the foundations of mathematics, it is used widely in a variety of fields including physics, engineering, and economics. It is a set of abstract mathematical concepts that have been developed over an extended amount of time, making it much more than just a technical tool [2]. The study of differential and cumulative rates of change for resolving complex dynamic systems and continuous change. In

ISSN 2959-6157

the 17th century, Leibniz and Newton invented calculus in succession. Leibniz provided a more condensed and comprehensive system of symbols from the standpoint of geometry, while Newton concentrated on using the "flow number method" to solve the problem of physical motion. Despite the differences in their methods, they both seek to answer issues of boundaries and change. The ideas and methods of infinitesimal calculus are indeed the product of a lengthy and nearly uninterrupted history of mathematical progress. Particular appreciation should be given to Newton's accomplishment of expanding and unifying the range of processes and Leibniz's connecting them with new methods of analysis and a new calculus [3].
This essay explores the significance of calculus in mathematics and applied disciplines, analyzes the significant contributions of Leibniz and Newton to the field's development, and discusses the function of the infinitesimal concept. The principal aim of this investigation is to explain the basic role of calculus in scientific research and to demonstrate its theoretical and practical significance.

2. Leibniz's Calculus

In the 17th century, Leibniz and Newton created the method of calculus nearly at the same time, Leibniz's differential and integral calculus and Newton's fluxional calculus, though different in many aspects, each involve a clear recognition of what is now called the inverse relationship between differentiation and integration [4]. However, Leibniz's differential and integral calculus weren't widely accepted by people in the 17th century, due to the political reason. Until he died several years later, his calculus method was gradually accepted by mathematicians.

Leibniz creates the symbol 'd' and '$\int$' to present differentiation and integration. To explain this, Fig.1 can be used to show the relationship between differentiation and integration.

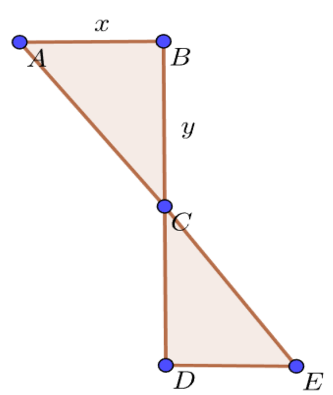

Fig. 1 Two similar triangles ABC and CDE, with an hourglass shaped [5].

Suppose AE is X and BD is Y, from the vertical angle of ACB and DCE, it can say that their tangents are equal to each other, indicating x/y equals to $X-x/Y-y$. Leibniz defines this as dx/dy, named differentiation, which means the difference between two unknowns, for instance, the Fig. 1. As the value of angle ABC decreases, x decreases and AE nearly coincides with line BD. When angle ABC is infinitely approached to 0, Leibniz suggests that instead of saying that the triangle has vanished, it should say that it has become unassignable while being perfectly determined. This is because, even if x and y are equal in this instance, the relation x/y is not equal to zero. After all, it is a perfectly determinable relation equal to dx/dy [3]. For these differentials, Leibniz gave the calculus rules and several examples of their uses. Two years later (1686a), he provided some guidance on the interpretation and application of the ∫-symbol. His innovative methods were not well received by the mathematical community quickly or productively because of how they were published. However, the calculus was approved [6].

3. Infinitesimal

Very small numbers technically are called infinitesimals [7].

$$1/2 > 1/3 > 1/4 ... > 1/m ... > [\delta] \quad (1)$$

δ is the symbol of infinitesimal, for any ordinary natural counting number m = 1, 2, 3, ⋯. It writes $a \approx b$ and says a is infinitely close to b if the number $b - a \approx 0$ is infinitesimal.
Infinitesimal can also be presented in Fig. 2.

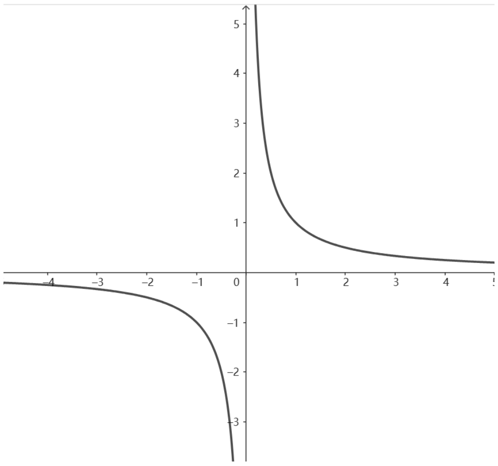

Fig. 2 An inverse proportional function (Picture credit: Original).

This is an inversely proportional function with the equation $y = 1/x$. From the graph, it is clear that when x increases, the y value infinitely approaches 0, the same as

x decreasing. When x is infinite, then y is $1/infinity$, which is infinitesimal. For example, if x is $1x108$, the y is $1/1x108$, which is $1x10-8$, a very small number, and define it as infinitesimal.

Calculus contributes a lot to the foundation of infinitesimal. However, the differential calculus was not based on using an infinitesimal and a summation of these quantities; rather, it was based on the notion that these infinitesimals were differences, his notation, the rules governing the notation, and the fact that differentiation was the opposite of a summation. Perhaps most importantly, the work did not require reference to a diagram.

4. Newton's fluxional Calculus

Newton's calculus was popular during the 19th century, however, nowadays, it is widely used in Physics rather than Mathematics. His calculus is published after Leibniz, and due to his political position and reputation in the academic circle, his calculus is considered more complete and advanced than Leibniz's calculus.

In Newton's "method of Fluxion", there's an effectiveness of the fluxion concept beyond the algorithmic level.

Now to this, I shall observe that all the difficulties hereof (the problems traditionally associated with the analytic art) may be reduced to these two problems only, which I shall propose, concerning a Space described by local Motion, anyhow accelerated or retarded.

I. The length of the space described being continually (that is, at all times) given; to find the velocity of the motion at any time proposed.

II. The velocity of the motion being continually given; to find the length of the Space described at any time proposed.

The first problem is solved by the tangent of the curve, which is the rate of change.

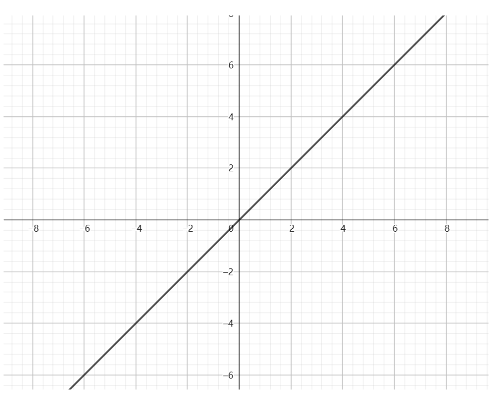

Fig. 3 A linear function $y = x$ (Picture credit: original)

Suppose the y-axis is the velocity and the x-axis is the time, and here is a v-t graph, whose equation is $y = x$ (Fig.3). The acceleration at $t = 4$ can be calculated as $v - u / t$ which is $4 - 0 / 4 - 0 = 1$. 1 is also the tangent of the line. In every value t, the tangent is always the acceleration. So, the first question, simply, can be solved by the tangent of the line about accelerated or retarded rate in motion. This is the predecessor of the fluxion method.

For Newton's calculus, it depends on infinitely small quantities, which is the velocity change of a variable, such as increasing or decreasing in time. Fluxions aren't important, instead, those ratios tend to be more significant.

Newton defines two words: fluents and fluxions, which means they are 'flowing quantity'.

Fluxtions: rate of change relative to time (e.g. dx / dt).

Fluents: variable such as x or y.

Compared to Leibniz's differentiation, Newton creates the 'dot-notation'. He presents dy/dx as ẏ/ẋ. For a single variable function, such as x, then its ? is 1.

Newton's fluxions provide a method for physicists to measure the variation of the position of the object when time increases. He treats the position as fluent, so the rate of change of position, which is the velocity, is fluxion. Nowadays, physicist usually uses differentiation and integration to calculate the velocity and acceleration. Newton presents them in the same way. Also, in Newton's book 'Principia Mathematica', he uses fluxions to calculate the law of motion of objects, such as a planet or projectile motion. Although the general method to solve motion problems still uses Leibniz's calculus analysis, in astronomy or gravitation, Newton's fluxion calculus is useful and more accurate to describe the change of rate. In all, Newton's calculus was crucial to the development of physics, while Leibniz's calculus played an important role in the development of mathematical analysis.

5. Conclusion

Both Newton and Leibniz created an infinitesimal calculus, and their work is widely used in every academic field, especially Mathematical analysis and Physics. It is hard to compare these two great mathematicians' accomplishments while they both contribute to their field.

From a Mathematical perspective, Leibniz's calculus seems more generous and convenient because dy / dx and $\int$ is easier to present, they simplify the process of differentiation and integration and make them more systematic. Leibniz emphasizes the importance of differentiation and integration, and they are always used in infinity and geometry. Compared to him, Newton's system of symbols (such as dot symbols for derivatives) is still used in phys-

ISSN 2959-6157

ics today, especially when describing motion and change. Newton's method is therefore better suited to dealing with time-dependent dynamic processes, which makes his calculus particularly effective in the field of physics.

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