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Comparative Analysis of Small Deflection Behavior in Cantilever Beams Using Structural Steel and Aluminum Alloy

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Abstract:

This research investigates the deflection change of cantilever beams made of structural steel (S355) and aluminium alloy (6061) under small deflection conditions. The main objective is to compare their performance in several aspects of load carrying capacity, deflection and safety systems through theoretical calculations and Finite Element Analysis (FEA). The research highlights that S355 exhibits higher stiffness and lower deflection compared to 6061 under the same load. However, 6061 demonstrates a higher safety factor, making it more suitable for applications where lightweight and safety margins are prioritized. The results provide insights into material selection for structural and mechanical applications, particularly where trade-offs between deflection and safety must be considered.

Keywords: Cantilever Beam, Small Deflection, Finite Element Analysis, material selection

1. Introduction

Cantilever beams represent one of the most common structural elements in mechanical and civil construction. They find wide applications in bridges, building structures, and lifting equipment. In all these fields, the quality of cantilever beam used is of utmost importance. There are several key indicators of cantilever quality. For example, the maximum load a cantilever can carry or the cantilever deflection to load ratio. Studying these contents is of great significance for optimizing the design of the beam, reducing material costs, and improving structural safety.

From a mathematical point of view, the relationship between the deflection and load of the cantilever beam can be calculated by theoretical formula. But due to the complex load conditions and material properties, it is difficult to directly calculate the real result from the theoretical formula. So another way is needed to verify it. Finite element analysis (FEA) is a mature numerical simulation tool that can effectively handle complex load and material nonlinear problems. And this just makes up for the shortcomings in the theoretical formula calculation. So in this article, both theoretical calculations and FEA will be used to verify the rationality of the results.

Structural Steel is a cantilever arm material widely used in common fields such as bridges, building structures and lifting equipment. Structural Steel has high strength, plasticity and relatively low cost, which is consistent with the strength and durability requirements of cantilever arms. However, for structural steel, the material properties of different structural steels vary greatly due to differences in specific processing methods and microstructures. In this paper, structural steel S355 is chosen as one of the cantilever materials. The yield strength of the S355 structural steel is 355 Mpa. [1]. The yield strength of Structural Steel S275 is 275 Mpa [1]. Compared with other structural steels, S355 has better strength and durability, so it is a better choice in most cantilever beam applications.

At the same time, Aluminum Alloy is also a very common material. It is often used in aircraft manufacturing, automobiles, building exterior walls, bridges and other fields. It performs well in weight and corrosion resistance. Among the large number of Aluminum Alloys, the most used material type is Aluminum Alloy 6061, which has good mechanical properties and processing properties. The yield strength of Aluminum Alloy 6061 is about 276 Mpa [2].

When a cantilever beam deforms, the relationship between deflection and load depends on many factors. The relationship between deflection and load can be categorized according to the material properties, structure and deflection amplitude of the cantilever beam. Two types of deflections can be classified by deflection small and large deflections. When under the condition of small deflection, the deformation amplitude of the cantilever beam deflection is relatively small for the length of the beam. In this case, the relationship between deflection and load is linear. The corresponding results can be calculated according to Euler-Bernoulli Beam Theory. When the load gradually increases and the deflection becomes larger relative to the length of the beam, it is necessary to consider the large deflection case. In the case of large deformation, Linear Elasticity Theory is no longer applicable [3]. Under large deflection, the relationship between deflection and load is nonlinear, and the effects of geometric nonlinearity and material nonlinearity need to be considered [4]. The results of large deflection are usually predicted by numerical solution [5-6]. In this case, Linear Elasticity Theory or FEA is usually used to calculate the result .

In real life, the analysis of small deflection is meaningful in different scenarios. For example, for the cantilever arm applied to the bridge, daily vehicles and pedestrians will cause the bridge to be in a state of small deflection [7]. Therefore, it is important to analyze the deflection of the bridge beam to ensure that it will not deform excessively during its service life.

Structural Steel and Aluminum Alloy are two materials widely used in modern engineering. This paper aims to analyze the performance of these two materials in cantilever beam structures by combining numerical simulation with theoretical calculation, especially the comparison of mechanical behaviors under small deflection conditions. The research results will provide engineers with a reference for material selection under different load conditions, help optimize the structural design of cantilever beams, reduce material waste and improve structural safety and durability.

2. Method

This paper will use the Structural Steel S355 and Aluminum Alloy 6061 mentioned in the previous paper as the materials of the cantilever beam. In this paper, it is assumed that the cantilever beam is solid and the material is of continuity and homogeneity. From the corresponding literature, it can be obtained that the density of Structural Steel S355 is 7850 Kgm^{-3} and the Young's modulus is 210 Gpa [1]. The Young's modulus of Aluminum 6061 is 68.9 Gpa [2]. I In this paper it is assumed that the cantilever beam is rigidly attached at one end. The free end is subjected to a concentrated load. And in this paper, it is assumed that the cross-section of the cantilever beam is 80 mm long, 80 mm wide and 1000 mm long. And it is assumed that the load at the free end is 1000 Kg .

The theoretical calculation method in this article adopts the calculation under the condition of small deflection. The $0 < (w \ddot{} ML) < 0.275$ is defined as small deflection range in this article [8]. In this formula, u represents deflection. L represents the whole length of the beam.

This article uses the Euler-Bernoulli beam theory to predict the bending of a beam under load. The beam theory assumes several key things. First, it is assumed that the shear strain is negligible compared to the bending strain. Second, it is assumed that the stress in the material is positively correlated with the strain. Then, since the deflection is small compared to the full length of the beam, it is assumed that the deflection is linear with the load. Finally, it is assumed that the cross section of the beam remains flat and normal to the midline, which means that the strain is purely bending and does not involve torsion or shear.

In the calculation of analytical solutions, from the Euler-Bernoulli beam theory we obtain [9].

$$k = \frac{d^2 w}{dx^2} = \frac{M(x)}{EI} \quad (1)$$

In equation (1), k refers to curvature of the beam, and w denotes the deflection of beam. The variable x indicates the distance from the fixed beam end to the point of the

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applied load. $M(x)$ represents the bending moment at a specific distance x from the fixed end. The symbol E stands for the Young's modulus, and I is the moment of inertia for the cross-section. The expression for the bending moment in equation (1) is given as follows.

$$M(x) = F(L - x) \tag{2}$$

The F refers to the concentrated load acting on the free end of the cantilever beam in the equation (2). L is the total length of the beam. When formula (2) is substituted into formula (1) it can be written as

$$k = \frac{d^2w}{dx^2} = \frac{F(L-x)}{EI} \tag{3}$$

In equation (3), deflection and load are successfully connected. Then, integrating equation (3) with respect to x gives

$$\frac{dw}{dx} = -\frac{F}{EI}\left(Lx - \frac{x^2}{2}\right) + C_1 \tag{4}$$

By integrating equation (4) again, we can obtain

$$w(x) = -\frac{F}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + C_1x + C_2 \tag{5}$$

Substitute the boundary condition into equation (5). When $x = 0$ at the fixed end, th deflection $w(x) = 0$ and slope= $\frac{dw}{dx} = 0$. Substituting into equations (4) and (5), we can obtain

$$c_1 = 0 c_2 = 0 \tag{6}$$

The final relationship between deflection and load is

$$w(x) = \frac{FL^3}{3EI} \tag{7}$$

Therefore, the analytical solution can be directly calculated from formula (7). The F is the known load. E is the known material property. The moment of inertia can be calculated as follows.

$$I = \frac{bh^3}{12} \tag{8}$$

The length and width of the square cross section are known for equation (8). Therefore, there are no unknown parameters on the right side of equation (7). Then it is possible to calculate the deflection of the cantilever beam. For the numerical solution, this paper chooses to use Autodesk Fusion software for modeling and uses the static stress module in the simulation of Autodesk Fusion for analysis.

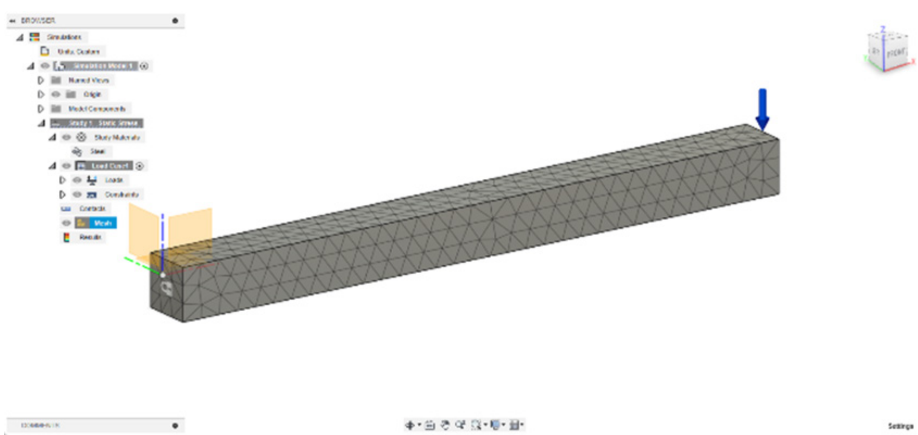

Fig.1 Meshed Cantilever beam

As shown in figure 1. What needs to be done is to add a constraint on the left side of the cantilever beam as a fixed end, and then add a vertical concentrated load on the rightmost side [10]. Then write the corresponding material properties. Then use the mesh function to decompose the cantilever beam into multiple small pieces. In this article, the 3% Model-based size in the software is selected to generate relatively small elements. This method can obtain more accurate numerical results. Then the computer can run the numerical solution.

3. Result

Table 1. Analytical solution results of Al6061 and S355

Material	Load(Kg)	Deflection(mm)
Al 6061	1000	13.8977
S355	1000	4.5664

The results of the analytical solution are shown as Table 1. The maximum deflection for AL 60601 is 13.8977 *mm* under a concentrated load of 1000 *kg* . The maximum deflection of S355 is 4.5664 *mm* . The above mentioned cases all belong to the range of small deflection because the ratio of deflection to cantilever beam length is less than 0.275.

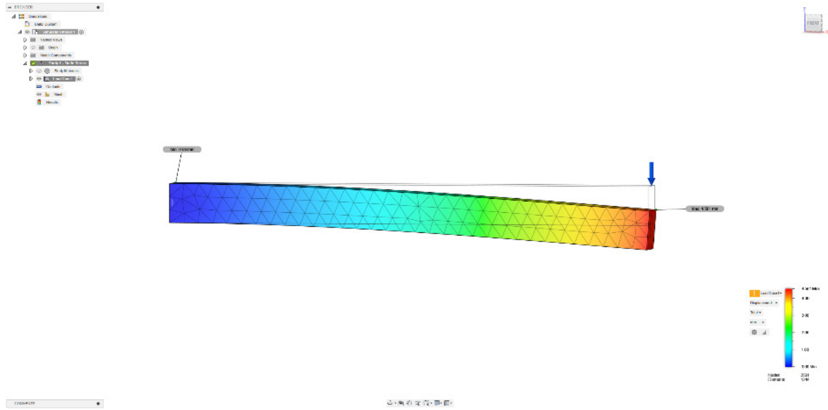

Fig.2 Deflection of S355

Figure 2 shows that the maximum deflection of steel S355 under a concentrated load of 1000kg is 4.61mm.

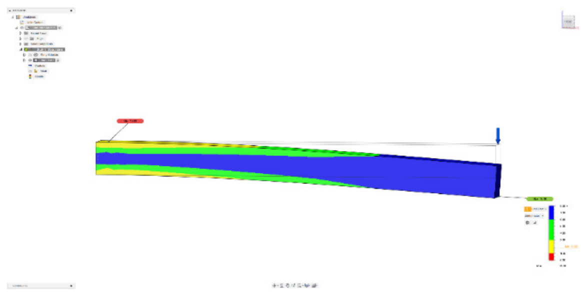

Fig. 3 safety factor of S355

Figure 3 shows that the minimum safety factor of S355 is 1.793.

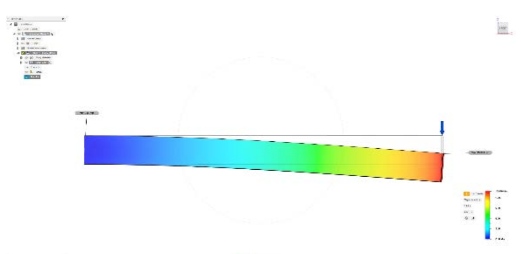

Fig. 4 Deflection of Al 6061

Figure 4 shows that the maximum deflection of steel Al6061 under a concentrated load of 1000kg is 13.879mm.

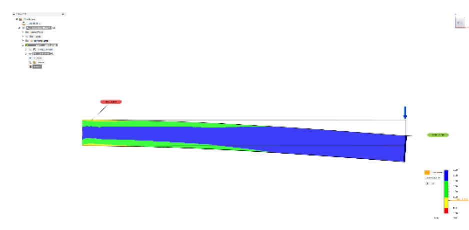

Fig. 5 safety factor of Al 6061

Figure 5 shows that the minimum safety factor of Al6061 is 2.366.

Table 2. Results of numerical solutions

material	Load/Kg	Deflection/mm	Safety factor
Al 6061	1000	13.879	2.366
S355	1000	4.61	1.793

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Table 2 summarizes the deflection and safety factor of the numerical solutions obtained in the previous figure.

4. Discussion

By comparing the results of analytical solutions and numerical solutions in Table 1 and Table 2, this paper finds that the results of the two are highly similar. This strongly proves the correctness of Euler–Bernoulli beam theory in small deflection deformation.

At the same time, Table 1 shows that when the cantilever beams of two different materials are subjected to the same concentrated load, Al 6061 produces a much larger deflection than S355. According to the analytical solution method, it can be inferred that this is because Al 6061 has a much smaller Young's modulus than S355. At the same time, this shows that under the condition of small deflection and from the perspective of deflection, S 355 is a better choice as a material.

Table 2 shows that the safety factor of Al 6061 is 2.366, while the safety factor of S355 is 1.793. Under the same concentrated load, the safety factor of AL 6061 is larger than that of S355.

material	Load/Kg	Deflection/mm	Safety factor
Al 6061	1000	13.879	2.366
S355	1000	4.61	1.793

From the perspective of calculation, this is because AL6061 has a higher yield strength than S355. This means that under the same load conditions, choosing Al 6061 as the material can provide more room for error in the design uncertainty. If the actual performance of some materials or structures is lower than the theoretical value or the load exceeds expectations, the safety factor can reduce the possibility of system or structure failure.

In summary, although aluminum alloy has a large deflection, it can still maintain the safety of the structure under large deformation due to its high safety factor. S355 has a small deformation due to its high Young's modulus, but a low safety factor, indicating that it has a small margin when bearing the ultimate load. Therefore, if the load is less than the yield strength of the material and the structure is required to maintain a small deformation, S355 is a better choice. If the deformation tolerance is high and a large load can occur, Al 6061 is a better choice as the cantilever arm.

In addition, although the paper compares the deflection performance of the two materials under the same load, it does not conduct a more extensive analysis of the performance under different conditions, such as comparisons under different loads or different geometric conditions. In the results section, more comparisons of different loads or different materials can be introduced, especially the analysis of other commonly used materials of the same type. This can not only enhance the breadth of the research, but also provide more convincing arguments. In the discussion section, provide more content based on actual engineering. For example, combined with the design standards of bridges or aviation structures, discuss the specific impact of material performance differences on practical applications, and explore whether there are improved design methods.

In the current study, the model of the cantilever beam was simplified to a beam with a rectangular cross-section and analyzed under idealized stress conditions. This simplification is effective for basic analysis and preliminary result verification, but in actual engineering applications, the model of the cantilever beam is usually more complex, and such simplification may not fully reflect the mechanical behavior of the real structure. Therefore, the simplification of the model brings some limitations and is difficult to apply to more complex engineering scenarios. In future studies, cantilever beams with complex cross-sectional shapes, such as I-shaped cross-sections, can be used to optimize structural performance. Secondly, cantilever beams often bear the combined effects of multiple complex loads, not just idealized concentrated loads. In bridges, buildings or mechanical structures, cantilever beams may bear distributed loads, impact loads.

Although the finite element analysis of the two materials was mentioned in this paper, there may be a lack of discussion on important details such as meshing method, convergence test, boundary condition setting, etc. In future research, it can be improved to increase the details of the finite element analysis, such as mesh type, mesh size selection criteria, whether the convergence analysis meets the requirements.

5. Conclusion

Through this study, it is found that the analytical solution of Euler–Bernoulli beam theory in the case of small deflection is correct and accurate. At the same time, it was discovered that S355 and Al 6061 metals should be selected according to different practical application requirements. S355 and Al 6061 exhibit significantly different deflection and safety factor characteristics under the same load. S355, with its high stiffness and small deflection, is very suitable for use in structures that require precise control of deformation, while Al 6061, because of its high safety factor, is suitable for lightweight structures that tolerate large deformations and require a high safety margin. Under design. And the difference in deflection and safety factor between S355 and Al 6061 is mainly attributed to the difference in their Young's modulus and yield strength. This study provides a basis for engineers to select Al 6061 and S355 as cantilever beam materials, especially under small deflection conditions. At the same time, using cantilever beams as an example, it can also be used as a basis for the selection of these two materials in any other field.

The current research faces the problems of insufficient application, insufficient assumptions about reality, and simple FEA methods. We will delve deeper into these issues in subsequent research and enhance the practicality of this research in the engineering field.

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