

# Analysis of elastohydrodynamic lubrication of ball screw under the influence of load

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## Abstract:

The lubricating oil film is crucial for the efficient operation of ball screws, as it significantly influences the adhesion and fatigue failure of the contact surfaces. This paper presents a comprehensive study on isothermal elliptical contact elastohydrodynamic lubrication (EHL) by developing a detailed model that examines the behavior of the oil film under various initial load conditions. Through rigorous analysis, we investigate the oil film pressure distribution and thickness variations, which are essential for understanding the lubrication performance in ball screw applications. Our findings reveal the nuanced relationship between load conditions and lubrication characteristics, providing valuable insights that can enhance the reliability and longevity of ball screw systems in engineering applications. This study aims to the broader realm of tribology through addressing critical lubrication challenges.

**Keywords:** elastohydrodynamic lubrication; elliptical contact; oil film thickness; oil film pressure

## 1. Introduction

The working state of the ball screw is closely related to the lubrication situation. In the process of ball screw failure, it often appears in the change from point contact elastohydrodynamic lubrication to exhausted oil lubrication. Consequently, investigating the EHL in point contacts holds considerable importance. To effectively analyze EHL, it is essential to integrate a variety of complex equations that account for fluid lubrication, solid surface elastic deformation, lubricant viscosity, density, pressure, and load. The interaction of these factors constructs a nonlinear system that poses important challenges in solving analytically.

Recent advancements in the field provide a robust foundation for understanding these complexities. R.

Gohar [1] and A. Cameron [2] successfully measured the oil film shape of Point Contact EHL. H. S. Cheng [3] presents Grubin's approximate solution for EHL. B. Hamrock and D. Dowson [4] offered an equation for the evaluation of film thickness of elliptical contact. C.H. Venner [5] established the calculation method of point and line contact pair lubrication under steady-state isothermal condition.

In a prior study [6], the isothermal EHL problem in elliptical point contacts between two surfaces with arbitrary velocities was effectively and directly resolved. As a result, a mathematical model has been established for isothermal EHL, the velocities at the surface can be directed concordantly, oppositely, or oriented at any angle in relation to the minor axis of the ellipse, considering non-Newtonian lubricants

in elliptical contacts. The isothermal EHL in ball screws under elliptical contact conditions with a rotating nut was investigated [7]. A numerical solution has been derived using a multigrid solver for various parameters.

VB. Awati [8] examined steady-state isothermal EHL of elliptical contacts, utilizing the Roelands pressure-viscosity model under changing rotation rates. It investigated the Reynolds and film thickness equations related to elliptical contacts within rolling bearings utilizing a hierarchical grid method with a complete approximation strategy. Dimensionless pressure distributions were derived, and equations for calculating film thickness in these contacts were formulated. Additionally, we developed friction models tailored for lubricants in the contact areas. To address the increased abrasion of the screw raceway caused by partial lubrication regimes in slow-speed ball screws, developed an EHL contact model for ball screws relied on the principles of isothermal EHL in order to examine the friction processes as well as wear behaviors at junction of the ball and raceway surfaces [9]. Study [10] employed a combined experimental and a numerical technique in order to analyze lubricant film thickness as well as frictional behavior within a broad elliptical EHL contact under sliding conditions. These conditions ranged from complete rolling to reverse sliding, with the slide-to-roll ratio systematically varied between 0 and 4.

Building on this existing literature, this paper utilizes a numerical iterative method to solve the isothermal elliptical contact EHL in steady state. By doing so, we aim to provide a detailed analysis of the impact of different loads on oil film thickness as well as oil film pressure, which are critical parameters for assessing lubrication performance. Our approach involves discretizing the controlling equations and applying edge conditions that mimic real-world operational scenarios.

Research result of this research will not only deepen our comprehension of lubrication mechanisms in ball screws. but also contribute to the optimization of design parameters in engineering applications. As load conditions vary, understanding how these changes affect lubrication performance is paramount for improving the reliability and efficiency of mechanical systems. Future work could extend the current investigation by investigating the effect of different lubricant formulations or surface treatments on EHL performance, offering further insights into enhancing lubrication efficacy.

## 2.Theoretical analysis

### 2.1 The equations of elliptical contact EHL

Supposing the lubricating oil in the problem to be solved is Newtonian fluid, under isothermal conditions, EHL

within elliptical contact interfaces is considered. in stationary condition, Reynolds equation is formulated as follows:

$$\frac{\partial}{\partial x} \left( \frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{\eta} \frac{\partial p}{\partial y} \right) = u_s \frac{\partial(\rho h)}{\partial x} \quad (1)$$

where,  $p$  is the pressure;  $h$  represents the thickness of lubricant film;  $u_s$  denotes the mean velocity of the upper and lower surfaces along the x-axis, while  $\eta$  signifies the viscosity of the lubricant;  $\rho$  represents the lubricant density;  $x, y$  is the coordinate which are same directions of the main speed and vertical to the main speed.

The boundary states are:

At the inlet  $p(x_0, y) = 0$ ;

At the outlet  $p(x_e, y) = 0$  as well as  $\frac{\partial p(x_e, y)}{\partial x} = 0$ ;

At the 2 sides  $p|_{y=\pm 1} = 0$ .

The film thickness are able to be given:

$$h(x, y) = h_0 + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} + \frac{2}{\pi E} \iint_{\Omega} \frac{p(s, t)}{\sqrt{(x-s)^2 + (y-t)^2}} ds dt \quad (2)$$

where  $E$  represents the equivalent elastic modulus of 2 contact materials;  $R_x, R_y$  --represent the equivalent curvature radii along X axis and Y axis, respectively, for 2 surfaces;  $h_0$  denotes the central film thickness, which is influenced by the load equilibrium condition.

In this work, we employ the viscosity–pressure relationship introduced by Reynolds:

$$\eta = \eta_0 \exp \left\{ (\ln \eta_0 + 9.67) \left[ \left( 1 + \frac{p}{p_0} \right)^z - 1 \right] \right\} \quad (3)$$

when the pressure  $p$  equals zero,  $\eta_0$  represents the lubricant's viscosity.

In EHL computations, the density–pressure relationship most frequently utilized is

$$\rho = \rho_0 \left( 1 + \frac{0.6p}{1 + 1.7p} \right) \quad (4)$$

where  $\rho_0$  denotes the density of lubricant when the pressure  $p$  equals zero.

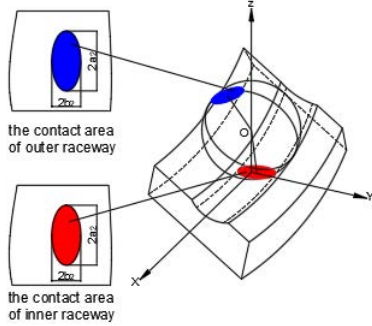
The hydrodynamic pressure generated in the elliptical contact region must support the applied normal load; therefore, force equilibrium equation can be given by

$$\int_{x_0}^{x_e} \int_{y_0}^{y_e} p(x, y) dx dy = \omega \quad (5)$$

### 2.2 The elliptical contact model of ball screw

When a load is applied, the initial contact point between rigid elements in a ball screw undergoes elastic deformation, resulting in the expansion of the contact area from a

point to an elliptical shape, as depicted in Fig. 1:



**Fig. 1 The diagram of elliptical contact of ball screw**

Let the coordinate axes of the contact principle plane be X and Y directions. The curvature of the ball on the contact point in O-XZ plane as well as O-YZ plane are  $R_{x1}$  and  $R_{x2}$  respectively. The curvature of the screw at the contact point within the O-XZ plane as well as the O-YZ plane are  $R_{y1}$  and  $R_{y2}$ , respectively. Consequently, the separation between the two rigid surfaces along the Z-axis is:

$$s = \frac{x^2}{2R_x} + \frac{y^2}{2R_y} \quad (6)$$

Among last equation:  $\frac{1}{R_x} = \frac{1}{R_{x1}} + \frac{1}{R_{x2}}$ ,  $\frac{1}{R_y} = \frac{1}{R_{y1}} + \frac{1}{R_{y2}}$ , then the synthetic curvature of the two contact bodies is

$$R = \frac{R_x R_y}{R_x + R_y}$$

Based on the numerical iterative method to solve the basic

equations of isothermal elliptical contact HEL, it is necessary to carry out finite difference discretization and dimensionless processing in sequence, simplifying the equations, and finally making the solution unrestricted by the unit. The variables employed in nondimensionalization

$$\text{include: } X = \frac{x}{a}, Y = \frac{y}{b}, e_k = \frac{b}{a}, P = \frac{p}{p_H}, H = \frac{R_x h}{a^2},$$

$$\rho^* = \frac{\rho}{\rho_0}, \eta^* = \frac{\eta}{\eta_0}, U = \frac{\eta_0 \mu_s}{ER_x}, W = \frac{\omega}{ER_x^2},$$

among which  $\omega$  means load per unit length as well as  $p_H$  represents Hertz contact pressure. After nondimensionalization, the equations set are able to be expressed as follows:

$$\frac{\partial}{\partial X} \left( \varepsilon \frac{\partial P}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \varepsilon \frac{\partial P}{\partial Y} \right) - \frac{\partial (\rho^* H)}{\partial X} = 0 \quad (7)$$

$$H(X, Y) = H_0 + \frac{X^2}{2} + e_k \frac{Y^2}{2} + P_{tr} \int_{x_0}^{x_e} \int_{y_0}^{y_e} \frac{P(S, T) dS dT}{\sqrt{(X-S)^2 + (Y-T)^2}} \quad (8)$$

$$\eta^* = \exp\{(\ln \eta_0 + 9.67)[(1 + \frac{P}{p_0})^z - 1]\} \quad (9)$$

$$\rho^* = 1 + \frac{0.6P \cdot p_H}{1 + 1.7P \cdot p_H} \quad (10)$$

$$\int_{x_0}^{x_e} \int_{y_0}^{y_e} P(X, Y) dX dY = \frac{2\pi}{3} e_k \quad (11)$$

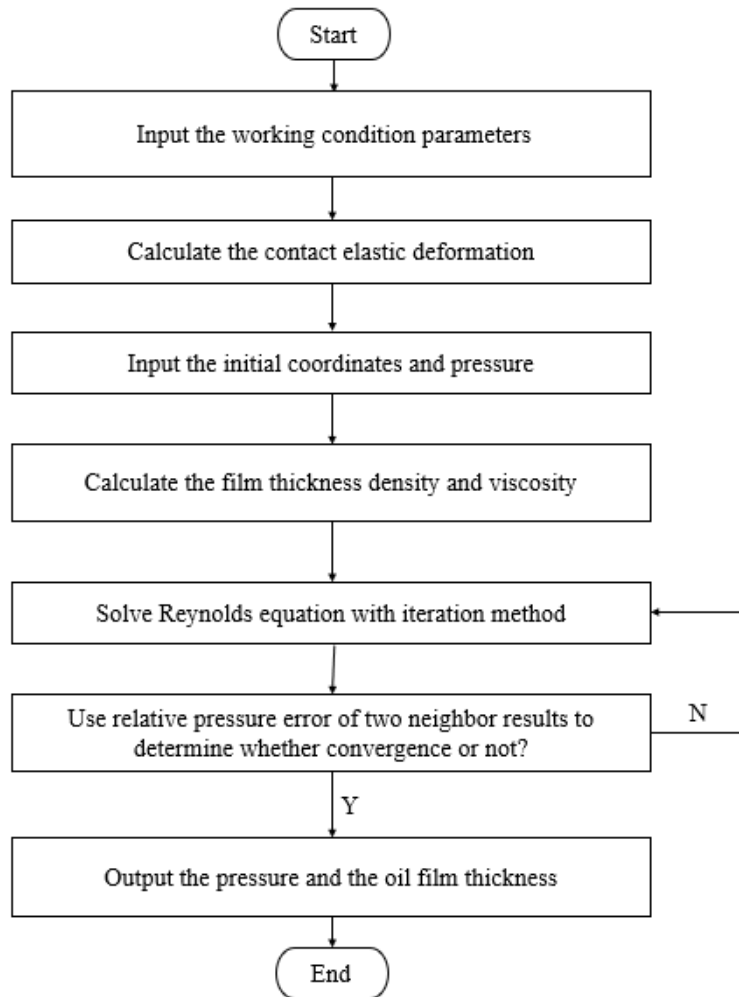
$$\text{Among this equation set: } \varepsilon = \frac{\rho^* H^3 b^3 p_H}{12 \eta_0 \mu_s R^2}, p_{tr} = \frac{3\omega R_x}{\pi^2 a^2 b E}$$

### 2.3 Calculation process

**Table 1 Parameters used in the simulation test**

Nodes amount, N×N	65×65
Nondimensional starting coordinate in X, X <sub>0</sub> direction	-2.5
Nondimensional ending coordinate in X, X <sub>e</sub> direction	1.5
Equivalent modulus of elasticity, E, (Pa)	2.21E11
Initial plastic viscosity, $\eta_0$ (Pa·s)	0.02
Initial density, $\rho_0$ (Kg/m <sup>3</sup> )	970
Equivalent curvature radii, R <sub>x</sub> (m), R <sub>y</sub> (m)	0.03, 0.06
Normal load, $\omega$ (N)	100.0 and 300.0
Average velocity, $u_s$ (m/s)	3.0

The computational graph for isothermal EHL within elliptical contact regions is illustrated in Fig. 2.

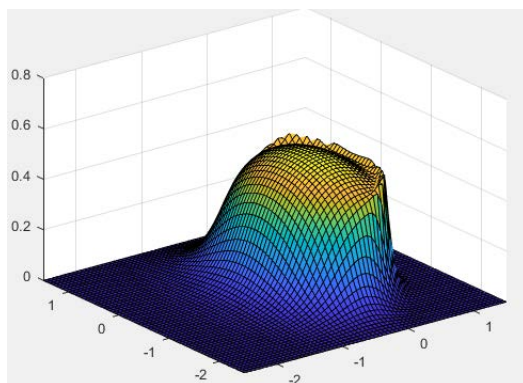


**Fig. 2 The computational diagram of isothermal EHL within elliptical contact interfaces.**

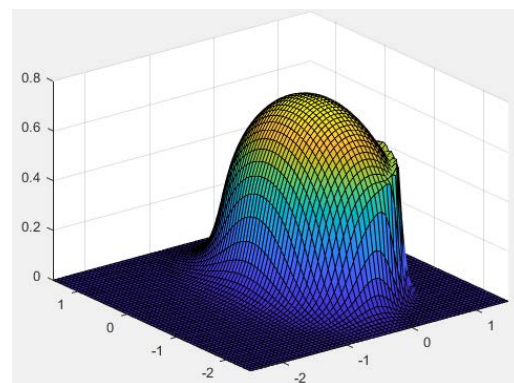
### 3.Results and discussion

Fig.3 and Fig.4.

Impacts of load parameter on oil film can be given in

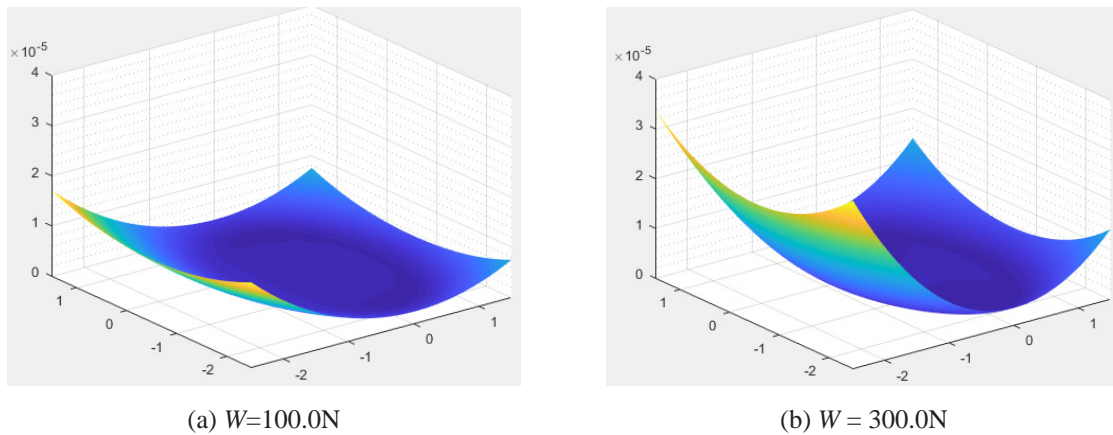


(a)  $W=100.0N$



(b)  $W = 300.0N$

**Fig. 3 Oil film pressure under the different load**



(a)  $W=100.0N$  (b)  $W = 300.0N$

**Fig. 4 Thickness of oil film under diverse load parameters:**

The analysis results indicate that the load condition exerts minimal impact on the thickness of the oil film within the context of isothermal elliptical contact EHL. Although the oil film thickness decreases slightly with the load increases, the change is relatively minor, suggesting that the lubrication film maintains a consistent thickness under varying operational pressures. In contrast, the effect of load conditions on oil film pressure is pronounced and warrants further discussion. As the load increases, both the primary pressure peak and the secondary pressure peak demonstrate a significant rise. Notably, the increase in the secondary pressure peak, while evident, is less pronounced compared to the primary pressure peak, indicating that the lubrication system's ability to sustain high pressures becomes increasingly important under heavier loads. This behavior underscores the critical role of hydrodynamic pressure in maintaining lubrication efficacy, which is vital for preventing wear and ensuring the longevity of ball screw systems. Understanding these dynamics can lead to improved design and optimization strategies in tribological applications.

We delve into the implications of our findings regarding the isothermal elliptical contact EHL model for ball screws. The observation that the oil film thickness remains relatively stable despite increasing loads suggests that the lubrication regime can sustain effective performance under varying operational conditions. However, the significant rise in oil film pressure with load highlights a critical aspect of lubrication management; maintaining adequate pressure is essential for preventing wear and ensuring the longevity of contact surfaces. This pressure behavior also points to the potential for optimizing lubricant formulations to enhance viscosity characteristics under higher loads. Additionally, the differences in pressure peak responses may offer opportunities for refining design parameters in ball screw systems, such as surface roughness or geometric configurations, to further mitigate the risks

of adhesion and fatigue failures. Overall, these insights reinforce the necessity of comprehensive lubrication analysis in the development and application of advanced ball screw mechanisms.

#### 4. Conclusion

In this paper, we established a comprehensive model of isothermal elliptical contact EHL specifically for ball screws. Our analysis focused on the oil film pressure distribution as well as the thickness of oil film under varying initial load conditions, providing critical insights into the lubrication dynamics involved. The research results present that while the load takes part in a minimal impact on the thickness of oil film—showing only slight decreases with increasing load—the effects on oil film pressure are markedly significant. As the load rises, we observed substantial increases in both the primary and secondary pressure peaks, highlighting the system's response to load changes. This distinction between the oil film thickness as well as pressure responses underscores the importance of hydrodynamic pressure in maintaining effective lubrication and preventing potential fatigue failures in ball screw applications. These findings provide valuable contributions to the domain of tribology and may inform future design improvements and operational strategies to enhance the performance and reliability of ball screws in various engineering contexts.

#### References

- [1] Gohar R, Cameron A 1963 Optional measurement of oil film thickness under elastohydrodynamic lubrication *Nature* 200 29-32.
- [2] Cameron A 1966 Theoretical and experimental studies of the oil film in lubricated point contact *Proc Roy Soc* 291 105-111.
- [3] Cheng H 1970 A numerical solution of the elastohydrodynamic film thickness in an elliptical contact *Journal of Lubrication Technology* 92 15-22.
- [4] Hamrock B, Dowson D 1976 Isothermal Elastohydrodynamic

Lubrication of Point Contacts: Part II-Ellipticity Parameter Results *Journal of Lubrication Technology* 98 375-381.

[5] Venner C 1991 Multilevel Solution of the EHL Line and Point Contact Problems: *Proefschrift*. 48-55

[6] Jiang M, Liu X, Yang P 2005 Analysis of Isothermal Non-Newtonian Elastohydrodynamic Lubrication of Elliptical Contacts with Two Arbitrary Surface Velocities *Tribology* 25 248-253.

[7] Mu S, Feng X 2012 Analysis of Elastohydrodynamic Lubrication of Ball Screw with Rotating Nut *Frontiers of Manufacturing and Design Science II, PTS 1-6* 121-126 3132-3139

[8] Awati VB, Naik S 2020 An Isothermal Elastohydrodynamic lubrication of elliptical contact with Multigrid method *Australian Journal of Mechanical Engineering* 18 375-384.

[9] Sun T, Wang M, Gao X, Zhao Y 2023 Study on the Friction Mechanism of the Ball Screws Based on the Isothermal Elastohydrodynamic Lubrication Theory *Journal of Beijing University of Technology* DOI: 10.11936/bjutxb2021080018.

[10] Amine G, Fillot N, Philippon D et al 2023 Dual experimental-numerical study of oil film thickness and friction in a wide elliptical TEHL contact: From pure rolling to opposite sliding *Tribology International* 184 DOI 10.1016/j.triboint.2023.108466