

Revealing the secret of dark matter through gravitational lensing

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Abstract:

This paper illustrates the major method of dark matter detecting, which is the gravitational lensing technique. Including details about the observational methods, analyzing techniques and dark matter mapping, this paper demonstrates the mechanism of gravitational lensing. Specifically, this paper illustrates the mechanism of gravitational shear, which is a method of concluding results of observations. Furthermore, convolving method is introduced, which helps form a comprehensive image and reduce the negative effects caused by noises. On top of that, the author has introduced an analyzing method based on the Bayes Theorem, which may be helpful for finding and abandoning unreasonable results. At the end, the author compares the differences and advantages of different analyzing methods, and gives examples of the major methods that are widely used in each field. This paper gives the future prospect of the field, including introduction to the edge cutting projects, references to relative dark matter theory, and several results that have already been found. For example, this paper raised the dark matter distribution of a galaxy taken by the Hubble Space Telescope as a representation for the development of dark matter study. Additionally, the author introduced the GRAVITY model that can analyze the results partially automatically.

Keywords: dark matter, gravitational lensing, observational techniques

1. Introduction

As Albert Einstein published his paper about general relativity in 1916, people realized that condensation of mass can deflect the space time. By observing the light that travels through the deflected space time, people can find new planets, galaxies and even dark matter. The technique of focusing on the curved space time is called gravitational lensing.

Since dark matter is thought to be unobservable, people cannot directly see the dark matter via photos or radiation detecting. However, with the help of gravitational lensing, it is possible for scientists to map the distribution of dark matter since it can still interact with gravitational force [1]. This unique property of dark matter, and the huge potential profits of studying it motivates the gravitational lensing technique to develop. By studying dark matter in the universe,

we can have a better point of view about how our galaxies are formed. The particles that form galaxies and clusters are thought to be closely related to dark matter, since dark matter particle can interact and produce those particles that form the universe. On top of that, dark matter keeps interacting with galaxies that have already been made, which means it can affect the evolution of galaxies as well. It is undoubtedly vital for us to study the evolution of galaxies, especially the Milky Way. Therefore, several dark matter detecting and analyzing projects have already been launched and some of them have already given some basic results about detecting dark matter. Another method for studying dark matter is n-body simulation, which offers a comprehensive way to study the evolution of galaxies, as well as its relationship to the dark matter. By combining the simulation and observation together, those various theory about dark matter can be tested. The most popular theory is called cold dark matter, but problems of the theory still exist, which need further simulation and observation to confirm whether the theory is appropriate or not.

When it comes to the observational part, detecting dark matter also needs huge amount of data analyzing efforts. Reading the observational results directly can be less effective and unprecise. Therefore, a various of data managing methods appears. For example, the results usually contain a significant percentage of noise, which is the useless information. By applying the Fourier transform to the results, only results of several frequencies that we need will be saved. On top of that, via introducing the Bayes theorem to those data, some results that are obviously incorrect can be abandoned. The statistics about previous observational results have formed the base of Bayes analysis method. The data analysis part in this paper will give an in-depth introduction to the conventional and modern methods including the two above. In the future, with the help of artificial intelligence, these methods can be applied in a more convenient way. In Shanghai, scientists have already created a model called GRAVITY that can distinguish unwanted information about microlensing images automatically. This model is still in testing step, but it can be seen as a step of development in data managing field.

2. Theoretical Framework

2.1 Gravitational Lensing

Nowadays, three main kinds of gravitational lensing called strong lensing, weak lensing and microlensing. Strong lensing implies that the planets in the picture may appear crooked shape due to some massive bodies such as

super massive blackholes. Weak lensing emphasizes the distortion on the picture. Microlensing is mainly about tiny differences that cannot be distinguished by human eyes. Weak lensing focuses only on the distribution of mass, regardless of the nature of mass and the dynamical state of the body [2]. Therefore, weak lensing matches the requirements of detecting dark matter, which makes it the main method for studying dark matter. As the author mentioned in the first paragraph, since the effects of non-uniform mass on the line of light cannot be ignored, the bodies on the pictures will have small distortion in their shapes, sizes and apparent brightnesses [3].

2.2 Dark Matter

Currently there is not a clear theory about dark matter, but some reliable models exist. A consensus is that dark matter shall include cold dark matter, which means before the universe is dominated by matter, the velocity of dark matter particles is not in ultra-relativistic. Cold dark matter (CDM) suggests that small particles with velocities much slower than the speed of light enables structural bodies to form, including planets and galaxies. Basically, the predictions that CDM and n-body simulation have made can match the results of observations. However, in several cases, some disagreements still cannot be ignored [4]. For instance, n-body simulation based on CDM suggests that the density of matter at the center of the Milky Way will increase subsequently (As the NFW distribution), when observation about dark matter dominated galaxies suggest that the density of matter at the center of them should be constant [5]. On the top of CDM, some scientists come up with the thought of warm dark matter (WDM), which indicates that dark matter particles have a velocity between relativistic and ultra-relativistic. The unique velocity allows dark matter particles to form structural bodies, without the problem of central density of CDM. However, WDM still have some problems of itself to be solved and studied [6].

3. Observational Techniques

3.1 Gravitational shearing

When it comes to the observation of dark matter, people emphasize mainly on two quantities, including gravitational shear and the convergence. Before introducing the two important quantities, a basic idea called distortion matrix is needed. Since people have the requirement of quantifying the effect of weak lensing, solving the lensing equation becomes vital. Defining a distortion operator called θ_i , which is extremely small that we only need to

emphasize the first order of its der Taylor Expansion [7]. Eventually, the lensing equation will be:

$$\theta_{s,j} = A_{ij}\theta_{i,j} \quad (1)$$

Here, i and j represent the i-th component and the j-th component of the lens plane and the source plane respectively. By using the Jacobian matrix of A, the convergence k and gravitational shear can be inferred:

$$A = (1-k) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \quad (2)$$

Here, k represents the enlargement of the background object in sizes, where indicates the stretches of it [8].

The gravitational shear (is the background distortion that exists in the universe, it is anisotropic and is correlated to two components γ_1 and γ_2 . For γ_1 , it is a field that describes the shear in x and y direction, when γ_2 describes the shear in $x=y$ and $x=-y$ direction [9]. In order to connect these components to the gravitational field strength, an equation is defined:

$$\gamma_1 = \frac{1}{2} \left[\frac{\partial^2 \Psi(\bar{\theta}_1)}{\partial \theta_{1,1}^2} - \frac{\partial^2 \Psi(\bar{\theta}_1)}{\partial \theta_{1,2}^2} \right] \quad (3)$$

$$\gamma_2 = \frac{\partial^2 \Psi(\bar{\theta}_1)}{\partial \theta_{1,1} \partial \theta_{1,2}}$$

An example can be used to show the relationship between gravitational shear and the universe. If the universe is a perfect circle with a radius of 1, the shear will make it an ellipsoid [10]. Another quantity that plays a crucial role in observation is convergence (k), which is isotropic. It can be related to the trace of distortion matrix A by: This equation builds up the relationship between k and gravitational field strength, the change in gravitational field strength is directly proportional to the projected mass density of the lens, which allows people to consider k as mass distribution [11].

4. Analytical Method

Analyzing the results of observation is the key to reveal the secrets of dark matter. During the observation of dark matter, the noise from the universe background has been a problem that must be solved effectively. On top of that, as the author mentioned before, though people can get data from the gravitational shear, there are still some steps needed to construct the final dark matter distribution map.

In this section, the author will introduce the method of the filtration of noise and mapping dark matter in detail

4.1 Convolution Method

A simple idea of filtration is to use convolving. There is a way of convolution using the Gaussian Window G, which has a standard deviation G. This method can suppress the signal with relatively high frequency, but the final signal can be confusing to read. Since the special property of convolution, the quality of signal depends highly on the value of G [12]. Though the Gaussian method still have some disadvantage, the simplicity of it makes it the standard and basic method of filtration. The improvement of the Gaussian method is called the Bayes updating method (BUD). By considering the information that people have already collected, the BUD enables the new signal to be more precise with the help of prior information. Basically, the BUD is to execute another step of substituting the signal into the Bayes Theorem equation, and it will produce a final signal considering the prior results.

The mapping is mainly about a technique called weak lensing local inversion. This method focuses on two main problems, and has unique advantages in solving these kinds of problems. The local inversion can work even if parts of information are lost, or the field source is finitely large. The equation of local inversion is as shown:

$$\frac{-1}{1-|g|^2} \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \begin{pmatrix} g_{1,1} + g_{2,2} \\ g_{2,1} - g_{1,2} \end{pmatrix} \equiv \nabla k \quad (4)$$

Where K represents $\log(1-k)$, the quantity k means convergence. By using the thought of Jacobian Metrix, for a finitely large field, there can be infinite number of formulae in order to reach idealistic precision. However, several drawbacks exist. The result of local inversion makes the geometry more complex, which add significant work to the subsequent analyzing processes. On top of that, the results will be more fragile in front of noise, which makes the signal less effective to be used.

5. Results

Currently, although the theory of weak lensing is still improving, there are some results that have already been found. The most significant one is the dark matter distribution in the clusters:



Fig.1 Dark matter map in Galaxy Cluster Abell 1689

Hubble Space Telescope ACS/WFC [10]

This is a map of dark matter pictured by the Hubble telescope from NASA. The cluster in the picture is called the Abell 1689, containing about 1000 galaxies and trillions of stars. The cluster is about 2.2 billion light year far from Earth. In order to figure out the dark matter distribution of it, scientists have observed 135 lensed pictures, with 42 galaxies offering the gravitational lensing effect. To paint the map of dark matter in the universe is not merely to show the results of observation, these maps can help scientists to further figuring out the evolution and growth of galaxies.

Nowadays by concluding previous observational results and combining baryons to the current profile, including the Navarro-Frenk-White profile (NFW profile). The baryon matter in the galaxies is thought to be separated in discs, gases and bulge components. Eventually, NFW profile can fit the results best. On top of that, results show that the conclusion basically depends on the discs baryon models, with minor effects from gas models. The best (NFW) model has local density of dark matter $\rho_{dm}(R = R_{\odot}) = 0.51 \pm 0.09 \text{ GeV cm}^{-3}$, and scale length $r_0 = 8.1 \pm 0.7 \text{ kpc}$, with $X_{min}^2 = 45.4$ [11].

6. Discussion

The study of dark matter is never the work of gravitation-

al lensing only, with the help of N-body simulation, the study about dark matter now has many branch ideas based on the current CDM theory. For example, a current study called “Fuzzy dark matter” (FDM) becomes popular. The FDM smartly combines astrophysics with quantum mechanics, and the predictions made by it can basically match the results of some observations and simulations. The gravitational lensing is a much simpler and cheaper way to study dark matter comparing to simulation, or theoretical study. However, some systematic drawbacks such as the inevitable noise in analyzing signals, the calculation needed to improve precision truly exist. But if we think inversely, the complexity in basic works such as calculation and filtration of signal makes gravitational lensing the method that can firstly benefit from the development of technology. With the help of the neural network technique, convolving work can be simpler; with advanced computers, the process of Gaussian method can be done more effectively. Therefore, studying dark matter through lensing is a vital method that have space for development, and it is worth for investing more.

7. Future Prospects

Currently, a group of scientists have already developed the technique of self-correction method. The advantage of this method is that it can check the observation results

using previous simulations and observations. The self-correction model can reduce the errors produced by massive bodies during the process of lensing. However, the current correction technology cannot fit in the high requirement of precision of observation. The remaining error is about 0.5, which is larger than the standard of 0.31 in cosmology. In the future, as the observational technique becomes more and more precise, the self-correction model still needs to improve.

As for microlensing, in the past few decades, scientists failed to distinguish two close images in microlensing pictures. The major problem of figuring out images in extremely precise scale have already been basically solved. In the European Southern Observatory, scientists are trying to use a new instrument called "GRAVITY" to distinguish microlensing images. The GRAVITY is based of interference light matrix, which is the standard method to solve this problem. Since the LIGO gravitational wave observatory has already discovered many images with multiple blackholes, in the future the new instrument can play an important role in analyzing these images.

8. Summary

The weak lensing process can be concluded into three steps: observe, standardize, analyze. By focusing on the curving effects produced by massive bodies, people can have a much wider view to the whole universe. Dark matter cannot be seen directly, but it still has strong gravitational effects that allows people to map it. In order to simplify the work, standardizing the background of the universe is vital. With the help of standard data of shear field and convergence field, measuring the lensing effects becomes an easier work. Analysing is a difficult step for any subject that related to signal, filtration of noise, increase the density of information are major and meaningful steps. In the future, the author believes that weak lensing will be developed into a new place more related to

Bayes Theorem, which means analyzing process will increase the importance of previous results, and apply them to produce more precise results.

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